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THE ROCKET COMBUSTION MOTOR

by Dr. Ing. Eugen Sanger

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1. Why Rockets?

Practical flight technique demands for certain special purposes an apparatus other than a propellor mechanism for the production of very high motive power for a short time only. For example, modern long-distance transport planes, due to their aerodynamic refinement, fly with such relatively weak motors that their take-off with slight reserve power is very long and troublesome. Similar starting difficulties have for a long time existed with powerfully-motored planes taking off from the water. In like fashion the velocity of ascent of pursuit machines with any given motor is very limited, nor can they practically climb high enough because a sufficiently powerful motor for ascent would be unnecessarily heavy for the task of flight and landing. Again, for certain flying performances (for example speed record flights) high powers of impulse are especially necessary. In this instance the high motive power is only usable through short intervals of time. In all these examples, the few seconds at the start or the few minutes necessary for rapid ascent or for speed records make demands which correspond fundamentally to those fulfilled by rocket motors. Their other far-reaching realms of application and their further development is known. *

2. The Jet Velocity.

Through the rapid expulsion of fuel mass m with effective velocity c opposite to the direction of flight during a short interval of time t, the rocket motor must exert on the flying mechanism a high thrust, $P = mc$. Therefore the greater c with a given thrust and time of operation, the smaller need be the total fuel carried, mt; similarly with a given fuel and a given thrust, the greater is the duration of the thrust. The primary demand on the rocket motor is therefore the greatest possible effective jet velocity. The introduction of rockets into flight technique has already become a serious question now that an exhaust velocity of $c = 3000$ meters per sec can be reached.

3. The Rocket Combustion Motor.

At present there is a way open for the attainment of high jet velocities. Combustion of fuel mixtures (fuel + oxygen) converts the heating value E in calories per kilogram of mixture into gases of high heat content

*H. Oberth, "Wege zur Raumschiffahrt", Munchen, 1929; R. Esnault-Pelterie, "L'Astronautique", Paris, 1930; E. Sanger, "Raketenflugtechnik". Munchen, 1933.

$$J_0 = \int c_p \, dT = n_0 E$$

and accordingly a high temperature T is developed in a combustion chamber capable of withstanding it; the gases of combustion then transform their heat into the energy of the exhaust jet according to the energy law of the transpiration of gases,

$$c^2/2g = n_d J_0/A$$

c_p is the specific heat in calories per kg of the exhaust gas at constant pressure, g in meters per second^2 is the acceleration of gravity, and A in calories per kg is the mechanical equivalent of heat. Neither of the energy changes follows completely according to the determined efficiency —the combustion with the chamber efficiency $n_0 = J_0 \, E$, and the exhaust with the nozzle efficiency

$$n_d = c^2/2g : J_0/A$$

The entire reaction occurs continuously and under a constant high pressure.

4. The Chamber Efficiency,

n_0 was investigated in a great number of model tests with oil-oxygen motors. The completeness of the change of energy E into J_0 and with it the chamber efficiency, is determined principally by the completeness of the combustion within the chamber. Other losses are unimportant in comparison with this—especially the loss of heat through the walls of the chamber when the propellants themselves are applied as coolants and arrive preheated to the combustion chamber.

The completeness of combustion depends on the thoroughness with which the propellants are mixed and the length of time they remain in the motor. The length of time they remain in the motor is proportioned between the time before and after ignition. The less the delay in ignition, the greater the burning time, accordingly it is best that the propellants arrive at the chamber pre-heated (perhaps thru use as cooling agents).

According to the results of the research which follow, the length of time during which the fuel remains in the chamber must be greater than approximately 1/500 second with a favorable state of operation. This depends chefly on the relation of the necessary chamber space V to the narrowest cross-section of the exhaust nozzle f'; on the other hand, it depends very little on the other conditions of operation such as the pressure of the exhaust gases or the like.*

The relationship between V/f' and n_0 is represented in figure 1, as far as may be ascertained from the tests carried out on a small scale (up to 30 kg thrust) large V/f' is desired so long as the chamber surface that is in contact with the flame does not grow too large, since in this case the heat absorbed by the propellants used as coolants can no longer be controlled and results in a dangerous condition. According to practical experience the first important rule of rocket motor construction becomes evident:

I. "The volume of necessary com-

*E. Sanger, "Neuere Ergebnisse der Raketenflugtechnik", Zeitschrift Flug. Wissenschaftl. Sonderheft. Wien, 1934.

bustion space (in cm^3) must bear to the area of the narrowest cross-section of the nozzle (in cm^2) the relation of from 50 to 5000 cm."

The best value of V/f' should be looked for somewhere in the vicinity of 500 cm. This best calue can be maintained in a motor cooled simply by its propellants only when the motor is relatively powerful — from 500 to 1000 kg thrust — since with these larger motors the relation of chamber wall area to chamber volume is sufficiently small to guarantee proper cooling of the chamber walls by means of the propellants that are to be burned in the chamber; that is, if too high a fuel pressure is not employed.

5. The Nozzle Efficiency,

n_d , signifies the degree of completeness with which the heat content J_0 of the gases is transformed into effective jet velocity carrying kinetic energy at the rate of $c^2/2g$. It is known that the effective jet velocity c is not identical with the true speed of transpiration of the exhaust jet* since it is derived from the effective thrust of the motor — which as a vectorial sum of the pressure of the gases on all the walls in contact with the combustion, is composed of the instantaneous sum of the impulses in respective nozzle mouth cross-sections plus the pressure of the exhaust gases on the nozzle mouth cross-section (effective thrust = impulse given by speed ahead + pressure impulse = collective thrust mc). For the determination of the nozzle efficiency the customary relation of the transformation of heat content into jet velocity in Laval nozzles is not applicable (for example, line a in figure 2). The effective velocity and therewith the efficiency, already very high with a small expansion ratio f/f' (f = cross-section of the mouth — figure 3) is larger because of the high gas pressures on the small orifice, and grows

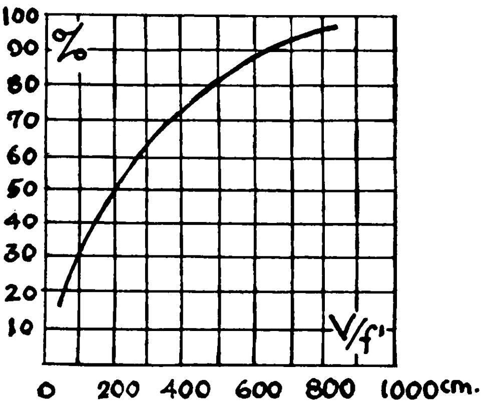


Figure 1. Chamber efficiency, no, plotted against the relation of combustion volume to nozzle throat cross-section

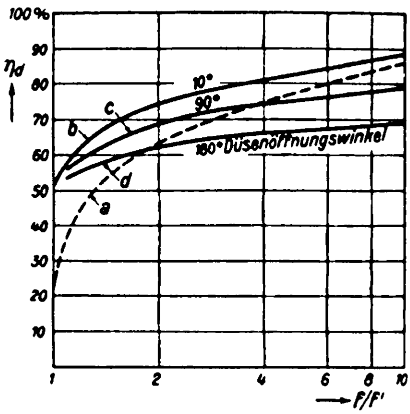


Figure 2. Nozzle efficiencies, nd, plotted against ratio of mouth area to throat cross-section (f/f' — expansion ratio)

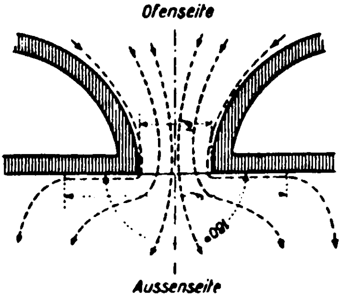


Figure 3. Expansion nozzle for the exhaust gases of a rocket motor having a 180 degree angle of aperture. (gases flow downwards).

with an increase in the expansion ratio, but not so rapidly as the true velocity of the escaping gases. The theoretical advantages of a greatly lengthened Laval nozzle of small angle of opening are therefore really less important for rocket motors than, for example, for steam and gas turbines.

Because of the small influence of the lengthened part of the nozzle, dispersion of the stream of gases from the nozzle wall beyond the narrowest cross-section of the nozzle, and the production of a similarly regulated and arranged stream of gases outside of the nozzle has, in great contrast to the turbine, only a slight significance. Therefore lengthened nozzles with great angles of aperture may be altered without perceptible damage to the nozzle efficiency. The nozzle angle can be, for example, 180^0 in which case the exhaust gases after emerging from the narrowed nozzle throat usually follow along the face wall, on which a pressure is accordingly exerted. Because of the decrease of pressure in the direction of the exhaust, the danger of the splitting off of the boundary coating of the nozzle wall is slight. Line d figure 2 shows that the effective nozzle velocity with much greater angles of aperture lies only slightly below those of very long Laval nozzles. Nozzles with flare angles considerably over 180^0 have practically no meaning. Short nozzles with great angles of aperture have the further important advantage of decreasing the surfaces endangered by heat and erosion. Thereupon follows the second important rule of rocket motor construction:

II. "Expansion nozzles for the exhaust gases of rocket motors should have an average angle of flare over 25^0 and under 270^0."

In practise the best value of the flare angle lies empirically somewhere around 90^0 (see figure 5). The nozzle efficiency in this arrangement is somewhat greater than with a flare angle of 180^0, but it does not quite reach the ordinary Laval nozzle with a 10^0 flare angle, as line c in figure 2 shows.

6. The Internal Efficiency.

n_i. The collective internal efficiency of the rocket motor

$$n_i = c^2/2g : E/A$$

is made up of the chamber efficiency and the nozzle efficiency

$$n_i = n_o \ n_d .$$

Empirically it can reach a value of 70%.

If one takes, for example, a certain fuel mixture — 1 part by weight of petroleum and 3.3 parts by weight of oxygen — there is obtained, with the calorific power of petroleum equal to

10,250 calories per kg, a total heat of reaction equal to 10,250/4.3 = 2390 calories per kg, and with this an attainable jet velocity of c = 3,740 meters per second, which has already very nearly been reached during the experiments with the small models mentioned previously. Since propellant mixtures with heats of reaction even up to 5,430 calories per kg are known, velocities of c = 5,650 meters per second appear attainable at least in theory. Operating safety limits us at present to jet velocities of 3,000 meters per second, corresponding to a propellant consumption of 3.3 to 3.5 kg of petroleum and oxygen per second of 1000 kg thrust.

7. Temperature of the Gases of Combustion.

The observed high chamber efficiencies are probably conditional upon the quickest possible combustion under a pressure of 20 to 100 atmospheres with no further decomposition of the gases, so that the flame temperature with oil and oxygen having a heat content of about

$J_0 = n_0 E = 0.85 \times 2390 =$ 2030 calories per kg must mount above ordinary combustion temperatures and reach a magnitude of the order of $T_0 = 6000^0$ absolute. Information obtained elsewhere under similar circumstances * points in the same direction as does also the magnitude of the heat loss thru the walls

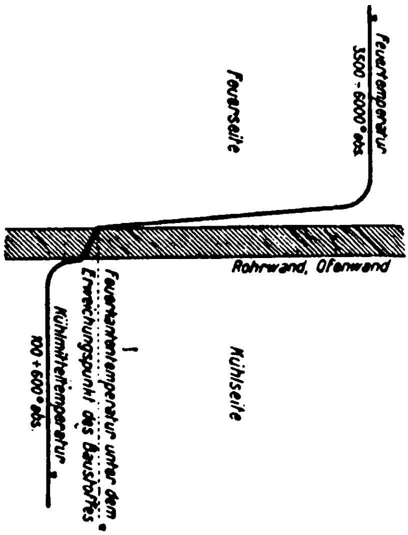


Figure 4. Temperature relations of the cooled chamber wall of a rocket motor.

of the chamber observed in the experiments. In any case the constant flame temperatures in the rocket combustion chamber exceeds those of any other type of high-temperature technical furnace, such as the internal combustion engine (2200^0 abs.), cannon (3000^0 abs.), or autogenetic welder (3500^0 abs.). The flame temperature can be reduced through the choice of a smaller V/f' (incomplete burning) or a fuel with a slight E value, however at the expense of the jet velocity. Structural solutions eliminate from the first any application of highly refractory lining of the chamber or of the nozzle without the essential wall cooling since the known substances with the highest melting points (graphite $400C^0$, tantulum hafnium carbide $390C^0$, niobium carbide $380 0^0$, thorium oxide 3000^0, etc.) do not reach with their melting points the temperature of the flame.

*Becker, "Physickalisches uber feste und flussige Sprengstoffe", Zeitschr. techn. Physik, 1922, No. 7.
*Geiger-Scheel, "Handbuch der Physik", Bd. XI, 1926, S. 369.
*Stettbacher, "Schiess- und Sprengstoffe", 1933, S.88-89.

8. Wall Cooling.

The following simple but fundamental experiment shows that one may protect, by means of cooling, ordinary materials of construction from extraordinary combustion temperatures: An ordinary metal tube (steel, copper, brass, aluminum, or the like) of about 10 mm. internal diameter is connected to a water pipe, and one tries to melt any thickness of it with an autogenetic welder while water is flowing through it. While the superficial poorly conducting layers burn off and melt quickly, the metal tube remains undamaged so long as the speed of the water inside it remains high enough. The metal tube is not even visibly warm. Since the welding flame temperature lies almost at 3500^0 abs. and the metal — which is not even glowing — is heated to only a few hundred degrees, the temperature drop in the border layers of the gases of combustion must have the extraordinarily high value of about 3000^0 abs. (figure 4).

Use is made of these relations in construction of the rocket combustion motor. Through a series of constructional precautions it is always possible to keep the temperature at the edge of a zone of extraordinary intense combustion low enough to be within the working range of the material of which the combustion chamber walls are constructed, providing the walls are not too thick and have good conductivity. Essentially, these precautions require a lowering to a minimum of the transfer of heat from the gases of combustion in contact with the wall surfaces, and a raising to a maximum of the transfer of heat from the cooler side of the wall to the coolant.

The supply of heat on the flameside of the wall comes principaly from the outflow of the gases, in contrast to which convection is unimportant. In the nozzle the temperature of the gases of combustion, and with its fourth power the outflow of the gases, decreases according to the law of conservation of energy,

$$\int c_p / A \times dT = c_x{}^2/2g$$

with an increasing jet velocity c_x; therefore the convection of the rapidly streaming gases increases rapidly and reaches a threateningly high value with a speed exceeding that of sound. Radiation is minimized thru such precautions as: the use of a slightly radiating, diathermous fuel gas (for example, H_2, H_2O, CO_2); silvering to get a brightly reflecting lining of the inner surface of the chamber; its spherical smooth construction; high combustion gas pressure permitting smaller chamber surface and nozzle cross-section with a given thrust — of course at the price of high pump power or heavy pressure tanks.

The removal of heat from the cooler side of the wall to the coolant takes place practically entirely thru convection. It is favored by precautions like the use of the coldest coolants (liquid gases) increasing of the surface in contact with the coolant (conduits and ribbed, rough wall surfaces) use of the densest possible coolant of vapors, gases, and liquids under high pressure (danger of va-

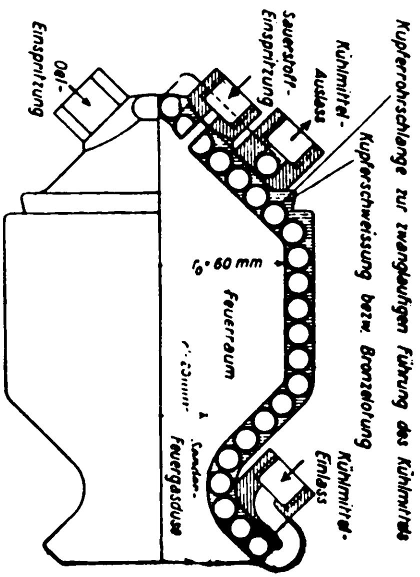


Figure 5. Design of a Sanger rocket motor for 500 kilograms thrust.

porization); high velocities of circulation behind even the smallest place on the wall surface in contact with the gases of combustion so as to raise the heat transmission value.

According to all experiments the ordinary circulation of the coolant in machines deriving their power from combustion is altogether insufficient in rocket motors, where the transmission of heat through the walls amounts to about 1 H.P. per square centimeter, and only with a strictly regulated and rapid circulation of the coolant behind the smallest part of the wall next to the fire can such quantities of heat, which are everywhere constant, be carried away without local transmission of heat in the chamber walls. This strong positive circulation of coolant is only possible in spaces of chiefly one dimensional extension (canals). Whereas in explosion motors, gas turbines, etc., local failing of the coolant to work, due to stagnation or too slow circulation, may be compensated for by heat flow within the walls without undue heating, this is not possible in a rocket motor working at maximum efficiency. In view of this the third important rule of rocket motor construction becomes evident:

III. "The circulation of the coolant along the walls in contact with the flame must take place in canals and must be so positive that it maintains unfailingly at every section of the wall in contact with the flame a velocity of circulation of a prescribed amount."

9. Structural Solution.

A structural solution of a rocket motor which observes these three fundamental laws of construction is shown in Figure 5. The chamber and nozzle consist of a coil of copper tubing which is made into a gas-tight, pressure-resisting wall by means of copper welding or bronze soldering. The coil of copper tubing through which water, oil, or liquid oxygen flows answers completely to the fundamental experiment described in section 8. The propellants are sprayed in continuously through interchangeable injection ports inclined at 45^0 to the axis of the motor. The motor operates at 20 atmospheres pressure and delivers an effective thrust of about 500 kg. It is planned only for experimental ground tests.

*E. Sanger, "Der Raketentrieb fur Flugzeuge", Zeitschr. Der Pilot, Heft 1, Wein, 1935.

10. Operating Characteristics in Flight

The fuel consumption of the rocket — 1.7 to 3.5 kg per sec per 1000 kg thrust — is very high in contrast to that of the ordinary propellor mechanism. The rocket therefore cannot enter into competition with a mechanism using the propellor as its continuous driving force. Rather, the rocket is fundamentally to be used as a source of impulse which lasts only a short time. The propellor mechanism in the more speedy airplanes yields something like 1 kilogram thrust for each kilogram of its own weight; the rocket motor in contrast yields 10 to 50 kg thrust per kg of its own weight. For equal weights the thrust of the rocket is therefore 10 to 50 times higher than that of the propellor mechanism. Hence follows the use of the rocket motor for an output of power over a short period of time.

The thrust of the propellor mechanism decreases with increasing speed of flight, since thrust times velocity is equal to a constant output. In contrast to this, the motive power of the rocket is quite independent of the velocity. The rocket motor is therefore decidedly the source of power for flight at high speeds.

Very real increases in the speed of flight are only possible in the stratosphere in altitudes over 20 km. into which no airplane can penetrate with power furnished by the screw action of the propellor. The propellor mechanism needs the surrounding air for its operation for three reasons — its oxygen content for the combustion of the benzines, its mass for the proper screw action of the propellor, and its ability to take up heat for the motor cooling — but the rocket motor takes care of all these functions with its own supply of propellants and does not therefore depend on the existence of the exterior atmosphere. Accordingly, just when the propellor mechanism, in altitudes over 15 km is weakening in spite of compressors and exhaust turbines, the rocket propulsor begins to display its advantages since its motive power is undiminished and it permits of the greatest possible speeds of flight. The realms of operation of both kinds of flying craft are therefore clearly separated by their physical limitations; the propellor mechanism as an economical source of impulse for speeds of flight under about 1000 km an hour in altitudes under 15 kilometers; the rocket motor as a source of maximum impulse for short times in starting, ascending, and reaching top flight speeds, above 1000 km an hour in altitudes above 15 to 20 km.

11. The Rocket Motor as an Accessory to Military Planes.

An example of the possibility of adapting the rocket motor economically is its use as an auxiliary to the existing propellor mechanism of pursuit planes.

A modern pursuit plane of about 1700 kg weight in flight, and capable of about 500 km per hour mean horizontal speed, climbs to an operating altitude of 6000 meters in about 8 minutes. By the effective use of a rocket accessory permitting this airplane with its speed of 500 km per hour (or 139 meters per sec) to climb with a favorable angle of ascent of 300, its operating altitude of 6000 meters would be reached in t = ($6000/\sin 30^{0}$) : 139 = 86 seconds, inclusive of time of starting — that is, in approximately 90 seconds or 1½ minutes. Under these conditions the propellor mechanism operating with compressed gas as in horizontal flight takes care of the thrust necessary to overcome air resistance (air resistance × forward speed) while the power necessary for ascent (weight × velocity of elevation) is provided wholly by the rocket accessory.

The installation of the rocket mechanism should be possible without important alterations in the airplane — particularly without modification of the present propellor mechanism, or without any appreciable increase in air resistance, even with airplanes already made, so that in this way the economical modernization of antiquated airplanes is possible. By feeding the propellants with high-pressure tanks the accessory is made completely independent of the main mechanism or of fuel pumps, but also since the propellant pressures are limited, the V/f' value must remain fairly low because of the relationship previously pointed out between the volume of the combustion chamber and the coolable wall surface, and this in turn lowers the jet velocity, which however, is unimportant for rocket motors used for this purpose (in contrast to the stratosphere rocket motors). The specific fuel consumption then amounts to about 3.5 kg sec t, so that the help in ascent during the 90 seconds for a motor delivering 1000 kilograms of thrust necessitates a propellant consumption of 3.5 × 90 = 315 kilograms of propellants.

The essential constitutents of the ascent auxiliary are the tank installation and the rocket motor. The tank construction must provide that the necessary propellants (oil vapor and liquid oxygen) are fed for complete combustion under a pressure of say 50 atmospheres.

The required pressure during the consumption of the propellants is maintained from a special compressed

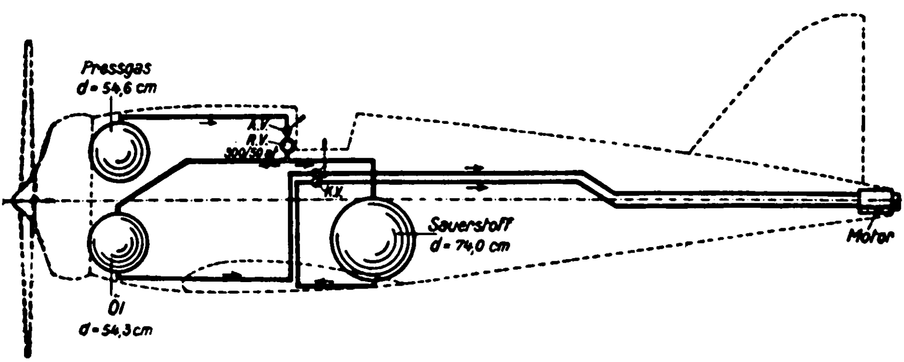


Figure 6. A rocket motor as an auxiliary in an ordinary pursuit plane.

gas container by way of a reducing valve. As a pressure-providing gas, nitrogen is used because of its neutral chemical properties, its insolubility in contact with liquid oxygen, and its cheapness.

Since the fuel oil and oxygen are used in a ratio of weights of 1:3.3, the necessary tank volume is 85 liters of oil and 215 liters of oxygen. The spherical high-pressure tanks are therefore of 55 cm and 74 cm interior diameter and 6.3 kg and 15.9 kg weight. The spherical pressure gas container, with its contents at 300 atmospheres pressure assuming the expansion of the gas due to the many changes of temperature, can be estimated at about 86 liters contents, 55 cm diameter, and 40 kg weight; accordingly the nitrogen gas weighs an additional 32 kilograms. This nitrogen gas is calculated as a material to be consumed. After the exhaustion of the supply of oil or oxygen it enters from the corresponding propellant tank into the motor, extinguishes the fire instantly, and flows thru the motor to the outside, in so doing completely washing out the motor after the cessation of thrust.

Figure 6 is a schematic arrangement of this ascent auxiliary which does not change the center of gravity of the pursuit plane in which it is installed.

The pressure gas flows from the pressure tank through a release valve and a reducing valve RV which keeps the pressure constant, then through a T fitting to the propellant tanks which are thus placed under 50 atmospheres pressure. The oil and liquid oxygen are forced by this pressure through the conduits leading to the motor, and are controlled by the coupled valves KV operated from the cockpit.

The total weight of the apparatus inclusive of mountings:

	Oil tank	7.5 kg
Empty	O2 tank	19.1
	Pressure tank	47.6 kg
	Motor	10.0 kg
Total		84.2 kg
	Oil	73.3 kg
	Oxygen	241.7 kg
	Nitrogen	32.2 kg
	Accessories	84.2 kg
Total		431.4 kg

The normal pursuit plane of 1700 kg weight has therefore a starting weight of 2130 kg after the ascent auxiliary is installed. By this means its starting speed is raised about 12% and the starting path is shortened about 50% by the influence of the rocket auxiliary. After using the rocket propellants the flying weight is only 1780 kg and during the period of ascent the total weight averages less than 2000 kg; and with this equipment the very high rocket thrust of 1000 kilograms is available through an angle of ascent of 30^0 — which is an achievement worth considering. The ascent auxiliary improves the time of ascent to 6000 meters from 8 min to 1½ min.

It would also be profitable to ascend through the lower layers of the air. where the propellor mechanism works well, without using the rocket auxiliary until 4000 meters. In this way the ascent to the upper air layers, which are difficult to reach, would be greatly accelerated. Or the ascent accessory can be repeatedly put into operation during the course of combat, for example in order to quickly regain lost altitude.

A flying craft so equipped is qualified to attack the world's speed record. In horizontal flight at 500 km per hour and air resistance of 2130/e = 2130/8.5 = 250 kg may be assumed for a pursuit plane of 2130 kg weight corresponding to a motor performance of 600 horsepower. After the ascent auxiliary is started and the plane is kept in horizontal flight with the propellor functioning also, the available thrust for overcoming air resistance mounts to about 1120 kilograms, or 1120/250 = 4.5 times the normal value, and the flight speed approaches $\sqrt{4.5}$ or 2.12 times this value; accordingly it reaches over 1000 km per hour and so surpasses the present world's record. This record speed would be maintained for a period of 90 seconds over a distance of 25 kilometers.

The requirements of the airplane cells do not expire above the degree that in any case is taken as the basis in the static reckoning of the pursuit plane. However, stability and the ability to control the airplane with motive power acting from the rear is in no way different than when the equally great motive power acts on the nose of the body, since the direction of the force remains in a fixed relation to the airplane axis and the point of exertion of the "fleeting line" vectors of force for the mechanical operation of the rigid airplane body is known to be without importance.

12. Cost of Operation.

The manufacturing costs for the motors plays an entirely subordinate role — for example, the cost of the complete ascent auxiliary should not lie over 2000 reichmarks, the life of the tank equipment is unlimited and that of the motor about equal to that of an ordinary airplane motor.

The price of oxygen can be placed at 0.50 marks per kilogram, the price of oil at 0.10 marks per kilogram. With a propellant consumption of 3.5 kg per second t, the cost of operation becomes 1.35 marks per second t. As an aid in starting the normal transport plane of 4000 kg starting weight a thrust of 1000 kg through about 20 seconds is required, so that the cost of operating amounts to about 30 marks per start together with the pressure gas. As an aid in ascent for the pursuit plane of 1700 kg starting weight a thrust of 1000 kg works through 90 seconds at which time the consumption of the pressure gases enters so that the total operating cost for each ascent becomes about 135 marks. Later stratosphere mail-carrying craft of say 3000 kg net weight plus a 500 kg cargo require, over a 5000 kilometer course, a variable thrust of 9000 kilograms through about 650 seconds during which the total fuel consumption amounts to about 12000 kg corresponding to a cost of 4800 marks for the 5000 km flight or about 2 marks per kilogram kilometer for the cost of transporting a cargo at a much higher speed than is possible today.

— **Eugen Sanger**

FUNDAMENTAL EQUATIONS OF ROCKET MOTION

Part I — Flight in Airless Space

While very thorough and complete analyses of the basic laws of rocket motion have been made by Goddard, Esnault-Pelterie, Oberth, Sanger, and others, their work is largely buried in abstruse technical treatises which are not generally available and which are principally in foreign languages. It has seemed desirable to resurrect and simplify certain of this material into an accessible and comprehensible form, with a view to clearing up some of the misconceptions which so frequently exist regarding the theory of rocket flight. Such is the purpose of the following elementary discussion, which is based on the work of Oberth and Scherschevsky.

Since most modern work on rockets is directed toward the development of meteorolgical rockets, only vertical flight will be considered. thus simplifying the calculations. As a starting point, consider a rocket in free space, subject to neither gravity nor air resistance. Its motion is governed by the law $M\,dV = -c\,dM$ where M = mass of rocket and fuel, V = rocket's velocity, and c = exhaust gas velocity relative to rocket, all in consistent units. This is really a force equation, $M\,dV$ being the force required to accelerate rocket and $-c\,dM$ being the kick of the escaping gas (negative, because dM represents a decrease of mass.)

Rearranging this and splitting M up into mass of empty rocket M_r and fuel mass M_f :

$$\frac{dV}{c} = -\frac{dM}{M_r + M_f} \qquad 1$$

Integrating:

$$\frac{V}{c} = -\log_e (M_r + M_f) + K$$

Since at the beginning of the flight $V = O$ and M_f = initial fuel mass M_{fo} :

$$K = \log_e (M_r + M_{fo})$$

Therefore:

$$V = c \log_e \left(\frac{M_r + M_{fo}}{M_r + M_f} \right) \qquad 2$$

which gives the velocity for any value of M_f during flight. The maximum velocity V_m occurs at end of firing period, when $M_f = O$. Therefore:

$$V_m = c \log_e \frac{M_r + M_{fo}}{M_r} =$$

$$c \log_e \left(1 + \frac{M_{fo}}{M_r} \right) \qquad 3$$

The final velocity thus varies directly with the exhaust velocity, and hence as the square root of the kinetic energy of the exhaust gases (for K.E. $= \frac{1}{2} m c^2$) For any given fuel, the final velocity varies as the square root of the thermal efficiency, since the K.E. is directly proportional to the latter. It will also be seen that V_m depends on the initial mass ratio $\frac{M_{fo}}{M_r}$ (see Fig. 1)

Now, as Mr. Africano has pointed out,* the efficiency of conversion of exhaust gas K.E. into rocket K.E.,

*See Astronautics No. 32, "The Velocity-Ratio Efficiency".

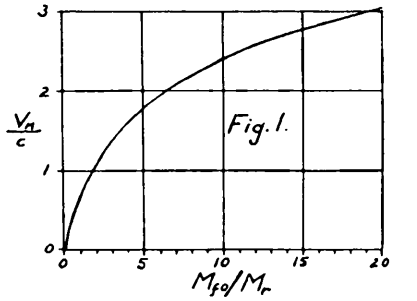


Fig. 1.

which is 100% when V = c, is less than 100% at higher or lower values of V. It is of interest to calculate the value of V_m for which the mean energy conversion is greatest.

The useful energy put into the rocket is the K.E. of the empty rocket at end of powered flight, which is $\frac{1}{2} M_r V^2$, while the K.E. imparted to the exhaust gases is $\frac{1}{2} M_{fo} c^2$. The ratio of these, the so-called "*energetic*" or "*ballistic efficiency*", is evidently:

$$E_{en} = \frac{\frac{1}{2} M_r V^2}{\frac{1}{2} M_{fo} c^2} = \frac{M_r}{M_{fo}} \left(\frac{V_m}{c} \right)^2$$

If we put in the value of V_m given by equation (3), we have

$$E_{en} = \frac{M_r}{M_{fo}} \left\{ \log_e \left\{ 1 + \frac{M_{fo}}{M_r} \right\} \right\}^2 \qquad (4)$$

By differentiating the above expression in respect to $\frac{M_{fo}}{M_r}$ and setting the result equal to zero, we obtain the equation:

$$\log_e 1 + \frac{M_{fo}}{M_r} = \frac{\frac{2 M_{fo}}{M_r}}{\left\{ 1 + \frac{M_{fo}}{M_r} \right\}}$$

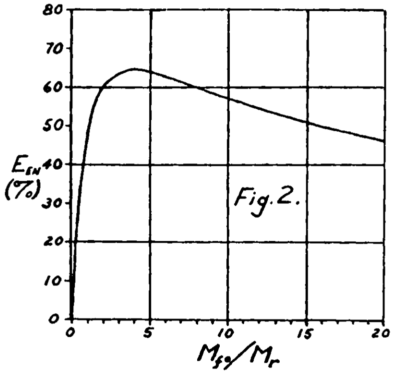
and when this is solved by trial-and-error, the result is:

$$\frac{M_{fo}}{M_r} = 3.9679, \text{ or very nearly } 4.0.$$

The corresponding value of E is 64.7%, which is the maximum value for the energetic efficiency under ideal conditions. It is impossible to obtain a more efficient energy conversion than this under any circumstances.

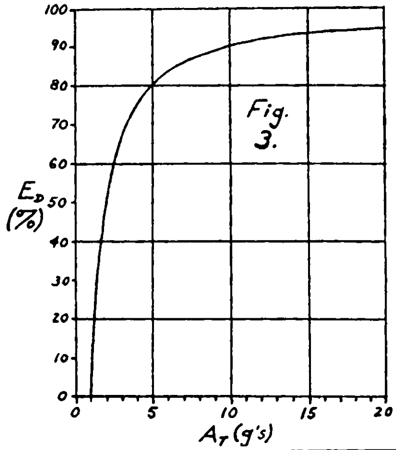
It will be noted that the form of the energetic efficiency curve differs considerably from Mr. Africano's velocity-ratio efficiency curve.* This is partly owing to the fact that the efficiency has been plotted against $\frac{M_{fo}}{M_r}$ instead of the velocity-ratio, thus giving a somewhat clearer idea of the efficiency, since taking the mean velocity-ratio efficiency as the criterion is only correct when the acceleration is constant. while $\frac{M_{fo}}{M_r}$ is independent of the form of the acceleration curve. Also, the final K.E. of the rocket is taken as the measure of useful work done (as assumed by Oberth, Sanger, and Scherschevsky) instead of taking the mean of the instantaneous values of the rocket's K.E. during the flight (as was done by Africano and Noordung). In this way the assumption of negative efficiencies, for values of $\frac{V}{c}$ higher than 2, is avoided. These negative values are apt to convey the false idea that all the fuel burnt after V has reached a value greater than 2c is "wasted" because the K.E. of the fuel remaining in the rocket is greater

*Astronautics No. 32.



than the chemical energy released if it is burnt and expelled from the rocket. However, the K.E. of the fuel in the rocket does not help to propel the latter. as burning the fuel and blowing it out would do. The situation is like that of a storekeeper obliged to sell his goods below cost; he incurs a loss, it is true, but not as great a one as if he had left the goods in his warehouse indefinitely, or given them away for nothing.

It will be seen from the graph that E_{en} gradually decreases for values of $\frac{M_{fo}}{M_r}$ greater than 4. This does not mean that there would be no point in building rockets lighter than this, as a glance at Fig. 1 will show. We are interested primarily in the absolute value of the rocket's performance, the velocity or altitude it will reach, and only secondarily in the relative efficiency with which the job is done. As a friend of the author's remarked, "It makes no difference if a thirty-mile altitude rocket has an efficiency of ½ of 1%; if it is the only device which will rise thirty miles, its efficiency for the pur-



pose is 100%." It is true that very high velocities can be obtained only by very light rockets carrying fuel loads out of all proportion to the pay load — thus a lunar rocket would probably be as big as a fair-sized battleship and carry an observation chamber about as big as the gondola of a stratosphere balloon, judging from calculations on the subject.

In the foregoing calculations, the effect of gravity has been neglected. Its action on the rocket is that of a force accelerating the rocket downward at the rate of 1 g (32.2 ft per sec per sec) while the thrust of the rocket propels it upward with an acceleration

$$A_T = \frac{\text{Thrust}}{\text{mass}} \times 32.2$$

the thrust and the mass being in similar units. The actual acceleration of the rocket is equal to the difference between these two accelerations:

$$A = A_T - g = \left\{ \frac{\text{Thrust}}{\text{mass}} \times g \right\} - g$$

$$= \left\{ \frac{\text{Thrust}}{\text{mass}} - 1 \right\} g$$

The ratio $\frac{A}{A_T}$ is the so-called *dynamic* or *accelerative* efficiency:

$$E_D = \frac{A_T - g}{A_T} = \left\{1 - \frac{g}{A_T}\right\}$$

(see Fig. 3) 5

Since the acceleration equals the instantaneous change in velocity,

$$dV = (A_T - g)dT = (A_T \; E_D)dT$$

$$V_m = \int_0^{V_m} dV = \int_0^T (A_T E_D)dT$$

When A_T and E can be written as integratable functions of the time, the integral can be reduced to a more direct form. Thus when E_D is a constant, which occurs when g/A_T is constant, we have simply:

$$V_m = E_D \, c \log \left\{1 + \frac{M_{f_o}}{M_r}\right\}$$

We must keep up a high mean value of E_D to obtain a high V_m that is, we must rapidly accelerate the rocket by means of a large thrust. Of course there are limits to this process; the larger the thrust, the heavier the motor must be, and the structure of the rocket must be stronger to take the heavier stresses. Moreover, delicate recording instruments may be seriously affected or even damaged by high acceleration. The most serious difficulty, however. is presented by air resistance. Obviously, if a rocket is to attain any considerable altitude it is advantageous to increase its speed only slowly at first, so that it can get through the thicker layers of the atmosphere without having its speed cut down too much by air resistance, and it should reach a high velocity only in the upper part of its trajectory, where the air is thin and the resistance small. But this means that the dynamic efficiency in the lower part of the flight will be very low, and much of the rocket's thrust will be wasted in pushing it slowly upward against the pull of gravity, instead of accelerating it and building up speed for the coasting period after it has ceased to fire. The selection of a compromise value for the acceleration which will best avoid these two evils is a difficult problem which has never yet been completely solved. In the case of powder-driven rockets, which do not attain an altitude sufficient for the change in air density to have much effect on the resistance, it is customary to use as high an acceleration as possible (15 or 20 g) and a large thrust and very short period of combustion. The more complicated question of the effect of air resistance on a high-altitude rocket will be taken up in Part II of this discussion.

— **J. H. Wyld**

Practical Rocketry, Aeroplane, June 17, 1936 — Review and discussion of Dr. Goddard's experiments.

Prize-Winning Paper on Rocket Design, Scientific American, October, 1936, by A. K. — General description of the A R S experimental technique and results achieved, discussion of REP-Hirsch award to A R S and to A. Africano. Two plates reproduced from "The Design of a Stratosphere Rocket."

MISCELLANIES OF ROCKETRY

Confirming the bulletin issued by the Secretary on July1, the following official communication from the President of the French Astronomical Society is printed regarding the award of the REP-Hirsch Prize for 1936:

"The prize has been distributed for the year 1936, *ex aequo* to the American Rocket Society and to one of its members, Mr. Africano. They have executed in 1934-35 careful experiments both at the proving stand and at rocket flights with the purpose of realizing a stratospheric rocket.

"Supplementarily the American Rocket Society has given these results extensive publicity by means of its journal. *Astronautics*.

"The French Astronomical Society wishes to express its keen regret that a very important work of Professor Goddard arrived so late that it was impossible to give him the reward which he merited for the exceptional value of his achievements.

"In addition, the French Astronomical Society desires to call attention to the interesting educational efforts of Mr. Alexandre Ananoff in connection with his lectures and publications."

This communication was contained in a letter dated June 22, 1936, from M. Andre Louis Hirsch, donor of the prize, to President Pendray of the A R S. M. Hirsch's letter concludes as follows: "I am happy to extend you my liveliest felicitations for your work, and to express the wish that you will continue to keep the French Astronomical Society informed of your researches, which have greatly interested our Committee."

The 2500 francs which is the Society's share of the award will be used for further experimental and educational work. Mr. Africano has informed the editor that he is devoting his share towards the expenses of a course at the Guggenheim School of Aeronautics (New York University) leading to a Master of Science degree with Rocket Research the subject of his thesis.

Mr. H. F. Pierce and Mr. Louis Goodman, members of the A R S, are constructing an experimental altitude rocket embodying several new principles in the design of its motor and its tank arrangement.

It was recently announced that Dr. Robert H. Goddard has received a further grant of $20,000 from the Daniel and Florence Guggenheim Foundation. The fund will finance a year's experimental work, which Dr. Goddard proposes to devote chiefly to the problem of weight reduction, now that a satisfactory stabilizer has been developed. He is also said to be investigating new light-weight and heat resistant alloys. It is his hope to evolve a powerful meteorological rocket ascending fifty miles or more and carrying automatic apparatus to record the solar spectrum and investigate electrical phenomena.

Mr. James H. Wyld member of the Experimental Committee of the A R S, this summer completed a trip to England during which he visited Mr. Phil Cleator, President of the British Interplanetary Society. First-hand information concerning the research technique of the A R S members was conveyed to Mr. Cleator to aid in formulating the British Society's experimental program.

The Society regrets the loss of one of its active members in the death of Miss Grace K. Morrison of Palm Beach, Florida, on September 5. Miss Morrison was active in aviation work and was the leading spirit in the creation of the Palm Beach Airport, which is classed as the fourth largest in the country. The port is to be named in her memory.

A new institution devoted to rocketry has been formed in Chicago. The "American Institute for Rocket Research", with C. W. McNash as president, announces its purpose to serve as a clearing house for information pertaining to this subject. An article by Mr. McNash in the August issue of *Popular Aviation* contains information concerning the plans of this organization.

A "Yale Rocket Club" is in the process of formation at Yale University. The organizer is Mr. Franklin Gates, active member of the A R S.

CURRENT BIBLIOGRAPHY

The Design of a Stratosphere Rocket by Alfred Africano, M.E., Vice-President, A R S. Journal of the Aeronautical Sciences, June, 1936 — Abstract of a thesis. Contains description of A R S proving stand tests and empirical motor formulas derived therefrom (as printed also in *Astronautics* 34), calculation of weight, fuel consumption, and altitude attainable of a rocket based on this test data, and performance efficiencies of such a rocket. Plates include typical motor test curves, layout of rocket, an improved motor design, and theoretical curves of rocket performance.

Les Fusees by Pierre Rousseau, La Nature, July 15, 1936 — General discussion of rocketry, resume of Dr. Goddard's report "Liquid Propellant Rocket Development." Plates reproduced from report and miscellaneous newsphotos.

The Rocket Propelled Aeroplane by W. Ley, Aircraft Engineering, Sept., 1936 — Discussion of the problems involved in the design and construction of rocket and jet-propelled aircraft. Account of experiments with powder-rocket gliders by the German Rhon-Rossitten Gessellschaft; and of the researches having to do with rocket aircraft of Melot, Goldau, Oberth, Goddard, and Sanger. Account of Greenwood Lake experiments. Illustrated by diagrams and photographs.

More Reviews of rocket literature appear on page 16.

FREE FALL AND THE HUMAN ORGANISM

The eventual development of manned rockets will require that human pilots endure without ill effects the condition of apparent 'weightlessness' that attends free projectile movement in space. There has been considerable conjecture as to how mind and body will react to this experience, and it is gratifying to find that the delayed parachute drop has been utilized by a competent medical observer to study this question.

Captain Harry G. Armstrong, M. D., Army Medical Corps, has used himself as a test case in such a jump at Wright Field, and his detailed report is printed in the *Journal of the American Medical Association* for October, 1935. Captain Armstrong allowed himself to drop for 11 seconds until he covered 1200 feet and ceased accelerating, when he opened his parachute.

His report of his reactions is distinctly encouraging. As to his mental conditions he declares, "Throughout the free fall, all conscious mental processes seemed normal. As soon as the airplane was cleared, fear and excitement disappeared.

"Consciousness was unclouded and ideation was rapid, precise, penetrating and clear." (This seems especially favorable from the point of view of the pilot of a manned rocket, who would of necessity keep his faculties clear.) "There were no consciously perceptible heartbeats or other bodily processes."

As to physical sensations, Dr. Armstrong writes, "The period of free fall was remarkably free from abnormal physical sensations. There was no nausea or vertigo, although I am quite susceptible to both from any swinging, tumbling motion, or disorientation. In this case the lack of a distinct sense of motion may have been a factor.

"There were no abnormalities noted in the cardio-vascular systems.

"There was none of the empty or "gone" feeling in the abdomen so common in elevators and airplanes.... Breathing was even, regular, and undisturbed."

While the duration of this experiment was of necessity quite short, and also strictly speaking "free fall" in the astronautical sense was only approximated because of air-resistance, nevertheless the results tend to remove at least one of the objections that have been raised against the rocket as a means of transport. Man will, it seems, be able to adapt himself even to the apparent absence of gravity.

(This effect will occur not only in falls towards the earth, but through all parts of a rocket trajectory in which the motor is not firing and air resistance is negligible. Weightlessness will be a phenomenon to take into account even for short flights from one part of the earth's surface to another, and not only for interplanetary voyages, of the sort described so graphically by writers.)

OPTICAL DETERMINATION OF JET VELOCITY

Supplementing his suggestion that the waves of dark and light flame reported in the June 2nd, 1935, tests of the Experimental Committee were sound effects and might be used to determine the jet velocity experimentally, Dr. George V. Slottman* calls attention to an article in the July, 1933, issue (Vol. 12, No. 7) of the *Journal of the American Welding Society,* on this interesting subject.

Dr. A. Hilpert, the author of the article on arc and gas cutting phenomena, describes the apparatus he used to measure the velocity and form of the jet. The essential requirements are a slow motion moving picture camera, an oscillograph, and a system of lenses and mirrors.

"The peculiar form of the beam," writes Dr. Hilpert, "which seems to be plaited, refers to standing oscillations in a beam moving with a velocity quicker than sound. The angle on which the plaited lines intersect —the so-called Mach's angle—makes it possible to calculate the velocity of the beam in every section."

This is evidence corroborating the general conclusion reached by the Experimental Committee that the jet velocities of most of the runs were equal to the speed of sound in the jet gases. In one run which exhibited this standing wave phenomena, the jet velocity must have exceeded the velocity of sound.

The method used by Dr. Hilpert and other investigators of high speed gas and vapor jets to render the various flow line and standing waves visible might be studied to advantage in designing apparatus to record optically the action of a rocket motor jet.

— A. A.

*Of the Applied Eng. Dep't., Air Reduction Sales Co.

ERRATA

March, 1936 (No. 33) Page 20, next paragraph from last. Read, "A rocket recently built by the American Rocket Society weighed 20 lbs and carried 2 pints of gasoline." Article as printed incorrectly reads "5 pints of gasoline."

The decimal point was omitted in Equation 3, page 3, of the June 1936, *Astronautics* (No. 34). The equation should read: Pc = .75 Pf.

An error in placing the decimal point was made in the figures given in Table I, page 5, June 1936, *Astronautics,* for the efficiency of the Armengaude-Lemale gas turbine jet. The weight of jet gases flowing per second should be .32 instead of .032, the jet reaction should be 40 instead of 4 lbs., and the resulting motor efficiency should be 54% instead of 5.4%.

— *A. Africano*