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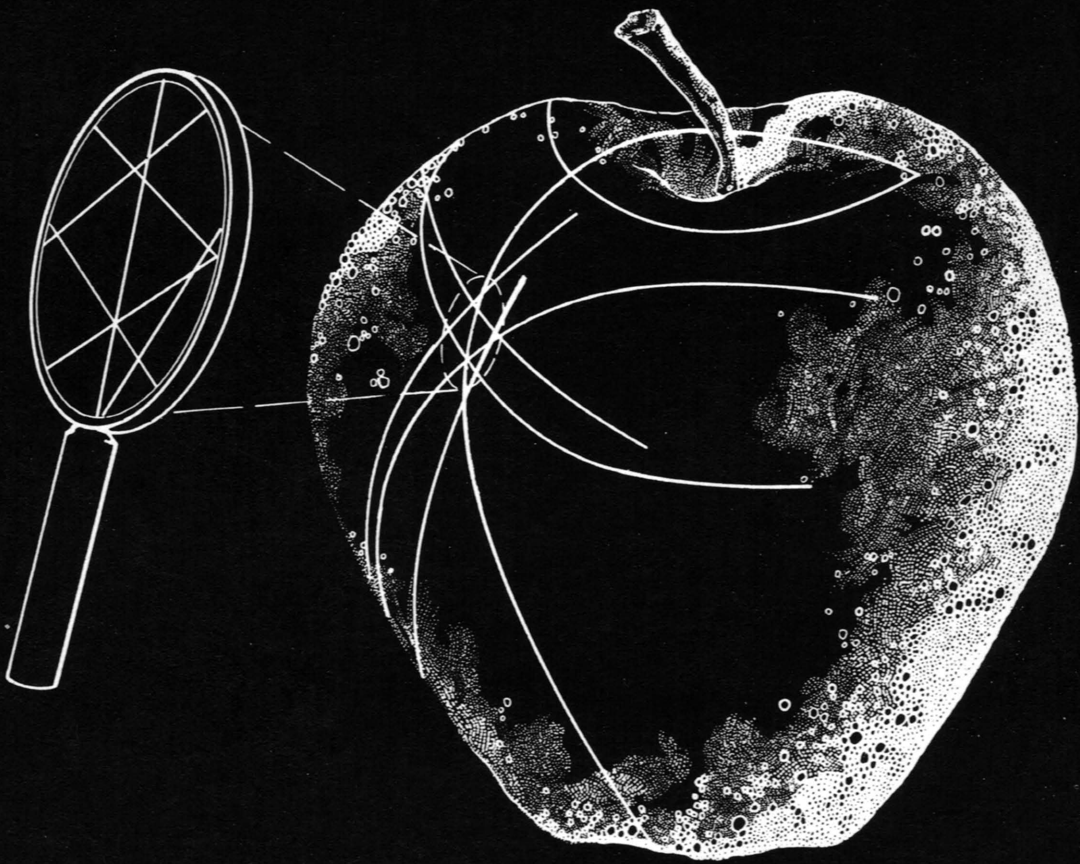

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CHAPTER 33

BLACK HOLES

A luminous star, of the same density as the Earth, and whose diameter should be two hundred and fifty times larger than that of the Sun, would not, in consequence of its attraction, allow any of its rays to arrive at us; it is therefore possible that the largest luminous bodies in the universe may, through this cause, be invisible.

P. S. LAPLACE (1798)

§33.1. WHY "BLACK HOLE"?

A dialog explaining why black holes deserve their name

Sagredus. What is all this talk about "black holes"? When an external observer watches a star collapse, he sees it implode with ever-increasing speed, until the relativistic stage is reached. Then it appears to slow down and become "frozen," just outside its horizon (gravitational radius). However long the observer waits, he never sees the star proceed further. How can one reasonably give the name "black hole" to such a frozen object, which never disappears from sight?

Salvatius. Let us take the name "black hole" apart. Consider first the blackness. Surely nothing can be blacker than a black hole. The very redshift that makes the collapsing star appear to freeze also makes it darken and become black. In the continuum approximation, where one ignores the discreteness of photons, the intensity of the radiation received by distant observers decreases exponentially as time passes, $L \propto \exp(-t/3\sqrt{3}M)$, with an exceedingly short e-folding time

$$\tau = 3\sqrt{3}M = (2.6 \times 10^{-5}\ \text{sec})(M/M_\odot).$$

Within a fraction of a second, the star is essentially black. Discreteness of photons makes it even blacker. The number of photons emitted before the star crosses its horizon is finite, so the exponential decay cannot continue

For a more detailed exposition of the foundations of "black-hole physics," see DeWitt and DeWitt (1973).

forever. Eventually—only $10^{-3}(M/M_{\odot})$ seconds after the star begins to dim (see exercise 32.2)—the last photon that will ever get out reaches the distant observers. Thereafter nothing emerges. The star is not merely "essentially black"; it is *"absolutely black."*

Sagredus. Agreed. But it is the word "hole" that concerns me, not "black." How can one possibly regard the name "hole" as appropriate for an object that hovers forever just outside its horizon. True, absence of light makes the object invisible. But couldn't one always see it by shining a flashlight onto its surface? And couldn't one always fly down to its surface in a rocket ship and scoop up a few of the star's baryons? After all, as seen from outside the baryons at its surface will never, never, never manage to fall into the horizon!

Salvatius. Your argument *sounds* persuasive. To test its validity, examine the collapse of a spherically symmetric system, using the ingoing Eddington-Finkelstein diagram of Figure 33.1. Let a family of external observers shine their flashlights onto the star's surface, as you have suggested. Let the surface of the star be carefully silvered so it reflects back all light that reaches it. Initially (low down in the spacetime diagram of Figure 33.1) the ingoing light beams

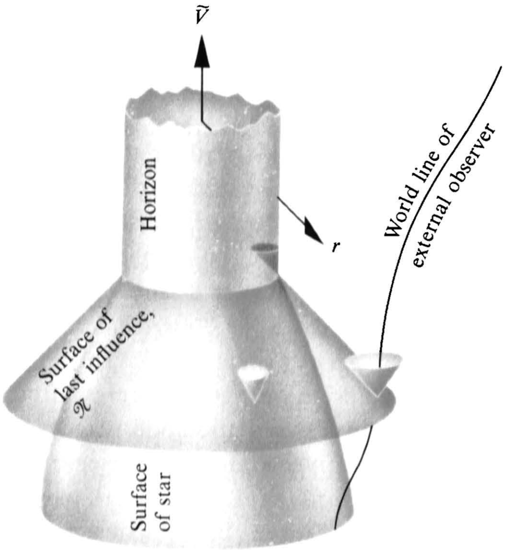

Figure 33.1.
Spherical gravitational collapse of a star to form a black hole, as viewed in ingoing Eddington-Finkelstein coordinates. The "surface of last influence," $\mathfrak{N}$, is an ingoing null surface that intersects the horizon in coincidence with the surface of the collapsing star. After an external observer, moving forward in time, has passed through the surface of last influence, he cannot interact with and influence the star before it plunges through the horizon. Thus, one can think of the surface of last influence as the "birthpoint" of the black hole. Before passing through this surface, the external observer can say his flashlight is probing the shape of a collapsing star; afterwards, he can regard his signals as probes of a black hole. For further discussion, see text.

reach the star's surface and are reflected back to the flashlights with no difficulty. But there is a critical point—an ingoing radial null surface $\mathcal{N}$—beyond which reflection is impossible. Photons emitted inward along $\mathcal{N}$ reach the star just as it is passing through its horizon. After reflection these photons fly "outward" along the horizon, remaining forever at $r = 2M$. Other photons, emitted inward after the flashlight has passed through $\mathcal{N}$, reach the surface of the star and are reflected only after the star is inside its horizon. Such photons can never return to the shining flashlights. Once inside the horizon, they can never escape. Thus, the total number of photons returned is finite and is subject to the same blackness decay law as is the intrinsic luminosity of the star. Moreover, *if the observers do not turn on their flashlights until after they pass through the null surface* $\mathcal{N}$, *they can never receive back any reflected photons!* Evidently, flashlights are of no help in seeing the "frozen star."

Sagredus. I cannot escape the logic of your argument. Nevertheless, seeing is not the only means of interacting with the frozen star. I have already suggested swooping down in a rocket ship and stealing a few baryons from its surface. Similarly, one might let matter fall radially inward onto the frozen star. When the matter hits the star's surface, its great kinetic energy of infall will be converted into heat and into outpouring radiation.

Salvatius. Thus it might seem at first sight. But examine again Figure 33.1. No swooping rocket ship and no infalling matter can move inward faster than a light ray. Thus, if the decision to swoop is made after the ship passes through the surface $\mathcal{N}$, the rocket ship has no possibility of reaching the star before it plunges through the horizon; the rocket and pilot cannot touch the star, sweep up baryons, and return to tell their tale. Similarly, infalling matter to the future of $\mathcal{N}$ can never hit the star's surface before passing through the horizon. The surface $\mathcal{N}$ is, in effect, a "surface of last influence." Once anybody or anything has passed through $\mathcal{N}$, he or it has no possibility whatever of influencing or interacting with the star in any way before it plunges through the horizon. *Thus, from a "causal" or "interaction" standpoint, the collapsing star becomes a hole in space at the surface* $\mathcal{N}$. This hole is not black at first. Radiation from the collapsing star still emerges after $\mathcal{N}$ because of finite light-propagation times, just as radiation still reaches us today from the hot big-bang explosion of the universe. But if an observer at radius $r \gg 2M$ waits for a time $2r$ after passing through $\mathcal{N}$ (time for $\mathcal{N}$ to reach horizon, plus time for rays emitted at $R \sim 3M$ to get back to observer), then he will see the newly formed hole begin to turn black; and within a time $\Delta t \sim (10^{-3}$ seconds$)(M/M_\odot)$ thereafter, it will be completely black.

Sagredus. You have convinced me. For all practical purposes the phrase "black hole" is an excellent description. The alternative phrases "frozen star" and "collapsed star," which I find in the pre-1969 physics literature, emphasize an optical-illusion aspect of the phenomenon. Attention must be directed away from the star that created the black hole, because beyond the surface of last influence one has no means to interact with that star. The star is irrelevant to the subsequent physics and astrophysics. Only the horizon and its external spacetime geometry are relevant for the future. Let us agree to call that horizon the "surface of a black hole," and its external geometry the "gravitational field of the black hole."

Salvatius. Agreed.

§33.2. THE GRAVITATIONAL AND ELECTROMAGNETIC FIELDS OF A BLACK HOLE

The collapse of an electrically neutral star endowed with spherical symmetry produces a spherical black hole with external gravitational field described by the Schwarzschild line element

$$ds^2 = -(1 - 2M/r)\,dt^2 + \frac{dr^2}{1 - 2M/r} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2). \qquad (33.1)$$

The surface of the black hole, i.e., the horizon, is located at $r = 2M =$ (gravitational radius). Only the region on and outside the black hole's surface, $r \geq 2M$, is relevant to external observers. Events inside the horizon can never influence the exterior.

The structure of a black hole is determined uniquely by its mass M, charge Q, and intrinsic angular momentum, S

The gravitational collapse of a realistic star (nonspherical, collapse with small but nonzero net charge of one sign or the other) produces a black hole somewhat different from the simple Schwarzschild hole. For collapse with small charge and small asymmetries, perturbation-theory calculations (Box 32.2) predict a final black hole with external field determined entirely by the mass M, charge Q, and intrinsic angular momentum S of the collapsing star. For fully relativistic collapse, with large asymmetries and possibly a large charge, the final black hole (if one forms) is also characterized uniquely by M, Q, and S. This is the conclusion that strongly suggests itself in 1972 from a set of powerful theorems described in Box 33.1.

Heuristic explanation of the M-Q-S uniqueness

Why M, Q, and S should be the complete governors of the final external field of the black hole, one can understand heuristically as follows. Of all quantities intrinsic to any isolated source of gravity and electromagnetism, only M, Q, and S possess (and are defined in terms of) *unique, conserved imprints* in the distant external fields of the source (conserved Gaussian flux integrals; see Box 19.1 and §20.2). When a star collapses to form a black hole, its distant external fields are forced to maintain unchanged the imprints of M, Q, and S. In effect, M, Q, and S provide anchors or constraints on the forms of the fields. Initially other constraints are produced by the distributions of mass, momentum, stress, charge, and current inside the star. But ultimately the star plunges through a horizon, cutting itself off causally from the external universe. (The nonpropagation of long-wavelength waves through curved spacetime plays a key role in this cutoff; see Box 32.2.) Subsequently, the only anchors remaining for the external fields are the conserved imprints of M, Q, and S. Consequently, the external fields quickly settle down into unique shapes corresponding to the given M, Q, and S. Of course, the settling down involves dynamic changes of the fields and an associated outflow of gravitational and electro-