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Chapter 4

JOHANNES KEPLER AND HIS LAWS OF PLANETARY MOTION*

Werner Schulz†

FROM KEPLER'S CURRICULUM VITAE

Three Hundred and Fifty years ago, on 15 November 1630, Johannes Kepler died at the age of 59 in Regensburg. On 8 October he had set out for Regensburg on horseback from Sagan in Silesia, his domicile while in the service of Albrecht von Wallenstein, with the intention of bringing before the Imperial Diet the fact that the emperor had neglected to pay him his salary, his dues amounting to a total of 12.000 guilders. However, the hardships of riding in bad weather along equally bad roads prevented him from doing so. Having been weakened so considerably he caught fever which led to his death within a very short time. For almost thirty years Kepler had been Imperial Mathematician, having served under the German Emperors Rudolph II (Emperor 1576-1612), Matthias (1612-1619), and Ferdinand II (1619-1637).

Kepler's life was full of hardships: a childhood spent in the atmosphere of a rather oppressive home of parents and grandparents, manhood overshadowed by intolerance due to the theological disputes between Lutherans, Calvinists and Catholics, and finally the hardships of the Thirty Years War.

Kepler's father had led an unbalanced life. Johannes Kepler (Figure 1), born on 27 December 1571 in Weil der Stadt in Würtemberg, was barely two years old when his father became a mercenary and went to fight in the Netherlands. His mother went along, and the child was left to his grandparents. In 1576 the parents returned, and the family moved to Leonberg, but one year later the father again became a mercenary, and when he finally returned in 1579, he had lost all he had acquired by providing bail for someone else. One may well say that the family life was rather chaotic.

Three times in his lifetime Kepler was driven from his home. From 1594 to 1600 he lived in Graz, where he had obtained his first position as a professor of mathematics at the protestant regional college, serving at the same time as regional

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† Deutsche Gesellschaft für Luft- und Raumfahrt (DGLR), Federal Republic of Germany. Dr. Schulz died in 1984.

mathematician for Styria. Being a Lutheran, however, he had to leave, when the Catholics took over again. He went to join Tycho Brahe (1546-1601) in Prague and was appointed Imperial Mathematician by Rudolph II after Brahe's death. When Rudolph died and Matthias became emperor, this appointment was confirmed, but the minimal salary Kepler had received so far was withdrawn altogether, thus forcing him to leave Prague. Having procured the emperor's permission and keeping his title as Imperial Mathematician, he moved to Linz, where he taught mathematics and philosophy. In 1625, however, all protestant preachers and teachers were expelled from Linz, and though Kepler as a civil servant of the imperial court did not actually fall under this decree, his library, which was essential for his studies, was closed down for some time. In the 1626 uprising protestant peasants besieged Linz and the printing house was destroyed, where the Rudolphine Tables Kepler had been working on for more than 25 years had been left to be printed. Kepler therefore left Linz, taking his family and all his material possessions with him. He travelled by ship up the river Danube to Regensburg. There he left his family, to continue his journey on his own by road to Ulm, where he succeeded, though not without difficulties, in having the Rudolphine Tables printed. Till 1628, when he moved to Sagan, he had no permanent home. Apart from Ulm, he had lived temporarily in Frankfurt, Regensburg, Linz and Prague.

Figure 1 Johannes Kepler (1571-1630).

In view of these hardships Kepler's achievements are all the more admirable. Kepler had received a good education despite the fact that he came from a poor family, for under the excellent educational system in Wurtemberg, gifted children of needy parents were sponsored by scholarships. Having finished primary school and Latin school at Leonberg with great success, Kepler was thus given the opportunity to attend the monastic schools at Adelberg and Maulbronn in preparation for his enrollment at the University of Tübingen in 1589. His intention was to become a theologian. Prior to the three years' course of studies of theology, however, in those days a two years' program of studies within the faculty of arts comprising the subjects philosophy, ancient philology, mathematics, and astronomy, was obligatory. Kepler's teacher in the fields of mathematics and astronomy was Michael Mästlin (1550-1631). Through Mästlin, Kepler was introduced to the ideas of Copernicus, but while Mästlin regarded the Copernican system only as speculation, Kepler very soon became convinced of its validity. In later reports he himself mentions that he frequently defended the teachings of Copernicus while still a student in Tübingen.

Shortly before finishing these studies, Kepler was faced with a decision which changed the whole course of his life, for when the post of a mathematics professor at the protestant college in Graz became vacant and the officials from Graz approached the authorities of the University of Tübingen to provide a successor, the senate of the University of Tübingen chose Kepler, possibly not solely because of his qualifications as a mathematician and scientist, but also because in view of his insufficient Lutheran orthodoxy he seemed less suitable for a clerical career. Kepler at this stage would have preferred to become a theologian, but as a grantee of the Duke of Würtemberg he was not in a position to turn down the offer from Graz. By taking up this teaching position in Graz, which at the same time comprised the writing of the annual calendar with its prognoses for the Styrians, his future career as astronomer was predestined. Soon, however, Kepler realized that he could serve God in this capacity no less than as a theologian. In a letter to his former teacher Mästlin, dated 3 October 1595 (KGW*, 13 (1945), 40), he wrote: "Having intended to become a theologian, I was rather disturbed for a while. Behold, for now you shall see, how God shall be praised through my efforts in astronomy."

KEPLER'S SCIENTIFIC WORK

A short essay is hardly the place to do justice to Kepler's scientific work in toto. It is thus concentrated on those aspects which are of particular relevance with regard to astronautics. These are, first and foremost, the laws of planetary motion.

In all, Kepler's publications number more than 80, and in addition he kept up a voluminous correspondence with a great many well-known contemporaries. About 400 letters from his and some 700 letters to him have survived, which allow a vivid insight into his way of living as well as into his way of thinking. As regards his publications, the most important ones for consideration are his three major works on astronomy: *Mysterium cosmographicum* ("The Mystery of the World")[3], his first

* Kepler's Gesammelte Werke (KGW), 22 volumes edited by W. von Dyck, M. Caspar, F. Hammer, M. List and C. H. Beck, München (1937-) [Ref. 14].

book written during the first years he spent teaching in Graz and published in 1596 in Tübingen, *Astronomia nova* ("New Astronomy")[6], compiled during the time he spent in Prague and published in 1609 in Heidelberg, and *Harmonices mundi libri V*[10*]) ("Five Books on the World's Harmony"), written in Linz and published there in 1619.

A popular version of Kepler's new astronomy appeared between 1618 and 1621. Intended for readers with a lower educational background it was published in seven volumes under the title: *Epitome astronomiae Copernicanae* ("An Outline of Copernican Astronomy")[9]. After the first part of it had appeared, the Vatican in Rome proscribed it[†]) in 1619, which had, however, apparently no effect on its demand, for already in 1635 a republication became necessary. The detailed new astronomy was first applied in the *Tabulae Rudolphinae* ("Rudolphine Tables")[12] (1627), a great tabular work begun jointly with Tycho Brahe on which Kepler spent, with interruptions, 26 years of his life. For more than a hundred years these tables served sailors all over the world as a basis for locating their positions.

Another work, which was only published four years after Kepler's death by his son Ludwig, also has a particular bearing on astronautics and space research. This is *Somnium seu Opus posthumum de astronomia lunari* ("A Dream, or posthumous Work on Lunar Astronomy")[13] (1634). Kepler wrote this work approximately at the time when his *Astronomia nova* went into printing. Later, when preparing the manuscript for publication, he added comments, which by far exceeded the length of the original text. By using the form of fiction Kepler demonstrates to his readers, how the motions of the planets, including the Earth, would appear to someone living on the Moon (something he initially probably wished to get clear in his own mind in order to grasp the Copernican astronomy).

In connection with astronautics, one should, furthermore, mention Kepler's comment upon receiving in 1610 Galileo Galilei's (1564-1642) publication *Nuncius sidereus* ("The Sidereal Messenger"), which represented the first report of the sensational discoveries made by telescope (mountains on the Moon, four satellites of Jupiter). Within a few days Kepler sent a most enthusiastic reply to Galilei, which he later published as *Dissertatio cum nuncio sidereo* ("Conversation with the Sidereal Messenger")[8] (1610) and in which he suggested that one ought only to invent ships and sails allowing for the navigation of the heavens in order to find men unafraid of the vastness of space: "Da naves, aut vela coelesti aurae accommoda, erunt qui ne ab illa quidem vastitate sibi mentuant." Astronautics foreseen by Kepler!

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* "Harmonices" is the genitive singular of "Harmonice".

† At the beginning, the Protestant Church too was opposed to the teachings of Copernicus. Martin Luther made some depreciating remarks about them in the early thirties of the 16th Century in his "Tischreden" (Table Talks), calling Copernicus a fool for insisting on the Earth being moved and not the heaven, since the Holy Bible says that Joshua ordered the Sun to stand still - and not the Earth. Philipp Melanchthon equally considered the idea of a moving Earth an attack against the Bible. In his book "Initia doctrinae physicae" ("The Foundations of Physics") (1550) he rejected the teachings of Copernicus.

The astronomer Kepler also concerned himself, as customary in those days, with questions of astrology. He was convinced that the stars had a bearing on men's lives, but he did differentiate. In a smaller publication *Tertius interveniens* ("The Intervening Third One")[7] (1610) he called consultative astrology the foolish daughter of the highly sophisticated mother astronomy and claimed for himself a third intervening position between those blindly believing in the stars and those denying all connections between the stars and human character. He considered it charlatanism, if someone pretended to be able to predict definite details of an individual's destiny from the constellation of the planets. When casting Wallenstein's horoscope in 1625, on the other hand, he explained that true astrology testified God's will and was holy and by no means to be taken lightly. His first task arriving in Graz was the compiling of the calendar with its prognoses for the year 1595 and since he succeeded by drawing careful conclusions from the political developments to make predictions, which turned out to be true--the Turk's invasion, the peasants' uprising--this calendar made him immediately famous in Graz.

Kepler's major contribution toward the sciences consisted in finding new approaches for theoretical astronomy. At least briefly, however, his achievements within other fields ought to be mentioned as well. Thus, he supplied new facts to the basic knowledge of optics and developed further the theories of light refraction and telescopic astronomical observation. He also improved the telescope. In mathematics, which to him was not an end in itself but a means, his achievements were equally important. Since calculus did not yet exist, he had to approach problems requiring this means (as, for instance, the derivation of his second law) by attempting to find solutions by appropriate methods of approximation. The invention of logarithms too was an event that only took place in Kepler's time. He immediately saw its utility and decided to make it available to future users of the Rudolphine Tables. As the tables of logarithms published by the Scotsman John Napier (Neper) gave no indication as to how one could calculate the logarithms, Kepler compiled a *Chilias logarithmorum* ("A Thousand Logarithms" - i.e. those of the number 100 to 100,000 advancing by always 100)[11] (1624) with a purely arithmetical explanation of the logarithm of his own which was based on infinitesimal derivations and the concept of limit.

Kepler also concerned himself with questions of applied mathematics. In 1615 he published in Linz a Latin work on a method to calculate the contents of barrels. A year later the German edition appeared intended for officials, merchants, landlords etc., in order to stimulate honesty in trade. While in Ulm in 1627 he proved himself of service to the governors of the city by systematizing the measurements that were in use there and by having cast a big iron kettle giving all the measurements for lengths, volumes and weights used in Ulm as an unmistakable and forgery-proof reference.

Kepler's three major astronomical works make no easy reading. This is mainly due to Kepler's style. Whereas in the case of Nicolas Copernicus' (1473-1543) one publication *De revolutionibus orbium celestium* ("On the Revolutions of the Heavenly Spheres") (1543), which discarded the idea of the Earth being the center of the universe, decades of laborious research had led to a style in which a sober matter-of-factness prevailed, and while later on Isaac Newton (1643-1727) in his *Philosophiae naturalis principia mathematica* ("Mathematical Principles of Natural Philosophy") (1687), with which he laid the foundations of classical mechanics, differentiated clearly between definitions, axioms, theorems, conclusions etc., Kepler laid open before his reader each process of thought. He did not restrict himself in his works to simply stating his final results, but even described in great detail those ways which had not led to the intended goal. Thus, his reader must follow him through a sort of maze, in which the right way is easily lost.

Kepler also felt no qualms expressing delight or disappointment in his scientific writings. Furthermore, he loved to introduce vivid comparisons in his writings as well as in his letters, and at times he showed a remarkable sense of humor.

MYSTERIUM COSMOGRAPHICUM

Coming to Graz as a young professor of mathematics, Kepler took an immediate interest in astronomy, and his official duties there left him sufficient time to indulge in his interest. Astronomy for him, however, did not only consist of observing the stars and calculating future positions of the planets. In his opinion, astronomy was not just a way of explaining how things were, but a means for deriving at the raison d'être of all things. Since God could not have created the world without an underlying purpose, and since man was not only endowed with senses but also with sense, it must, in Kepler's opinion, be possible to investigate the Creator's concept. Kepler applied himself to find out the reasons for the number of the planets and the size and the shape of their orbits:

> "What is the world, and for which purpose, according to which plan, was it created? Whence did He take the numbers and the norms He employed in His creation? Whence the number six in the system of the planets, whence the intervals between their orbits? Why is there such an enormous distance between Jupiter and Mars, which are not even the most distant of planets?"*

While pondering these questions in Graz, Kepler suddenly had, as he later reported, on 19 July 1595, in the middle of a lecture devoted to the problems of constellation, what to us nowadays appears to be a rather strange vision: It occurred to him that there was a relation between the six then known planets and the five regular Platonic polyhedra†) so that by nesting the planetary orbits and the regular polyhedra one into the other one might solve the "Mystery of the World". The considerations preceding this idea were as follows:

It occurred to Kepler that the radii of the planetary orbits as given by Copernicus showed a certain regularity. Taken in the order of Mercury, Venus, Earth,

* (Letter to Reimarus Ursus, 15 November 1595, KGW, 13 (1945), 48).

† There are exactly five regular polyhedra, i.e. bodies bounded by m equilateral n-gons: tetrahedron (m = 4, n = 3), hexahedron or cube (6, 4), octahedron (8, 3), dodecahedron (12, 3), and icosahedron (20, 3).

Mars, Jupiter, and Saturn, the radii approximately equal 4, 7, 10, 16, 52, and 95, if Earth's radius is taken to be 10, thus obeying a geometrical progression: 4 = 4; 7 = 4 + 3; 10 = 4 + 2x3; 16 = 4 + 4x3; 52 = 4 + 16x3; 95 ≈ 96 = 4 + 32x3. As this series could be continued infinitely, Kepler saw no reason why there should be only six and not 20 or 100 planetary orbits. He found himself at a dead end.

His next step was to interpose a planet between Mars and Jupiter as well as between Venus and Mercury. But this again did not lead him anywhere.

After a number of further attempts and having spend months calculating, Kepler finally hit upon the idea of the regular polyhedra. What he had in mind was this:

Take a spherical shell to represent Earth's orbit, and circumscribe this sphere by a dodecahedron, which again is to be circumscribed by a sphere representing Mars' orbit. This sphere is then to be circumscribed by a tetrahedron, which in turn is to be circumscribed by a sphere representing Jupiter's orbit. This sphere is to be circumscribed by a hexahedron (cube), which is to be circumscribed by still another sphere representing Saturn's orbit. Toward the other direction, inside the sphere representing Earth's orbit an icosahedron is to be inscribed, and inside this icosahedron another sphere representing Venus' orbit. And if one is to inscribe inside this sphere an octahedron and inside this octahedron another sphere, one arrives at Mercury's orbit. *Habes rationem numeri planetarum*.* ("And there you have the reason for the number of planets.")

In Kepler's opinion, this model was the answer to the question of the world's structure. The order of the regular polyhedra was given by the radii of the circumscribed and inscribed spheres. At the same time, Kepler believed that there were logical reasons for the order of the nesting, since one can differentiate between two groups of regular polyhedra. Cube, tetrahedron and dodecahedron form the first group which is characterized by the fact that from each corner issue three edges. In the case of the octahedron and the icosahedron, on the other hand, there are more than three edges to each corner. These bodies form the second group. The cube as the simplest three-dimensional shape must range highest, thus occupying the uppermost place with Saturn. Tetrahedron and dodecahedron belong to Jupiter and Mars respectively, and the two regular polyhedra of the second group are attached to the planets Mercury and Venus.

Kepler had still to take into account the fact the the planetary orbits according to Copernicus formed not concentric but eccentric circles. He did this by way of supposing for the spheres thicknesses corresponding to the eccentricities. Thus, his calculations finally resulted in figures for his model which more or less agreed with the values for the ratios of the planets' distances obtained by observation.

Even though Kepler's attempt to explain the structure of the world proved to be erroneous, his book is by no means worthless, for it contained a number of ideas

* (KGW, 1 (1938), 13).

which were of significance, and which he consequently followed up in his later works on astronomy. Thus, he investigated in Chapter 20 of *Mysterium cosmographicum* the question of what causes the planetary motions and attributes the fact that the planets move at different speeds to an *anima motrix* ("moving soul") within the Sun.

Kepler considered his book as a first revelation of his findings and an indication of what further investigations were to be carried out, as is shown in the title *Prodromus dissertationum cosmographicarum continens mysterium cosmographicum* ("Forerunner of Cosmographical Dissertations on the Mystery of the World") (1596)[5].

The appearance of this book soon made Kepler's name well-known among contemporary astronomers. For Kepler himself, the opinions expressed by Tycho Brahe and Galileo Galilei, the then leading astronomers to whom he sent copies of his book, counted most. Brahe appreciated Kepler's genius and his ability for abstract thinking, even though he was unable to agree with the *Mysterium cosmographicum* being on principle opposed to a theory *a priori*. Prerequisite, in his view, was an astronomy *a posteriori*, i.e. a mathematical theory based step for step on figures derived from observation. In spite of this, he invited Kepler to come to see him.* With Galilei, on the other hand, the differences in their way of thinking prevailed and prevented any fruitful contact.

Twenty Five years later, Kepler still did not consider this early work outdated. In 1621 he published a second revised and enlarged edition. An annotation to Chapter 20 refers to the *anima motrix*, saying that if one was to replace this word by *vis* (force), one would obtain just the principle on which celestial mechanics in *Astronomia nova* is based.

ASTRONOMIA NOVA

Kepler's official duties in Graz first prevented him from accepting Brahe's invitation, much as he would have liked to gain access to Brahe's observational data. Brahe, as was generally known, had for three decades concerned himself with carefully observing the positions of the planets and thus had at his disposal the data Kepler needed in order to pursue his studies to greater depths. It was only when Kepler was forced to leave Graz in 1600 that he joined Brahe in Prague. He became Brahe's assistant, and Brahe set him the task to formulate on the basis of the data gained by observation a theory for the orbit of the planet Mars, a task he himself and his assistant Longomontanus had failed at. In view of Kepler's own investigations this task was a most suitable one because of the fact that among the planetary orbits Mars' orbit shows the greatest eccentricity (apart from Mercury's orbit for which, however, due to its being so close to the Sun not sufficient observational data existed).

* (cf. Brahe's letter to Kepler, 1 April 1598 (KGW, 13 (1945), 197-202) and Mästlin, 21 April 1598 (KGW, 13 (1945), 204-205.

In order to be able to follow Kepler's way of approaching the problem, a few words must be said about the planetary theories which had been in existence from antiquity up to the time of Copernicus. Since Aristotle it was considered self-evident that a natural motion reverting to itself like a planetary orbit must be a uniform circular motion. The motions of the planets showed, however, irregularities in so far, as although the planets generally move forward each day in the ecliptic from west to east, they at times also move backward. It was therefore necessary to draw on auxiliary constructions such as epicycles and eccentric circles in order to be able to adhere to the theory of uniform circular orbits. Without going into detail, we can establish the following points: The center of the (circular) orbit of a planet and the center of the universe - the Earth according to Ptolemy - do not coincide. The line connecting the two points is the line of apsides from which the motion of the planet on its orbit appears uniform is the equant point. Ptolemy supposed the equant point to be symmetrical to the Earth with regard to the center of the circle (Figure 2).

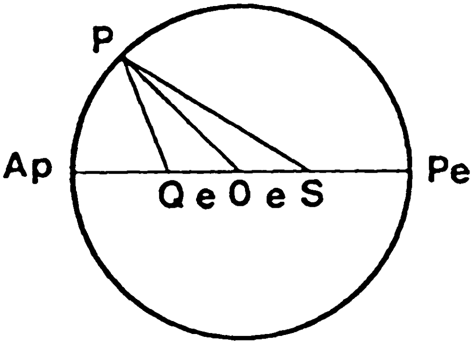

Figure 2 Eccentric circle: S = Sun, O = center of eccentric circle, Q = equant point, Ap = apogee, Pe = perigee, P = planet, e = eccentricity.

Copernicus also started out from eccentric circles, the difference being that he regarded the Sun and not, as Ptolemy, the Earth as the center of the universe. His calculations, however, did not refer to the Sun but to the center of the Earth's orbit. Like Ptolemy, he supposed equant points for the planetary orbits with the exception of Earth's orbit. With regard to the Earth he considered an equant point as unnecessary supposing the motion on the orbit to be uniform.

Kepler's task of developing a theory of the Martian motion consisted in calculating the line of apsides and the extent of the eccentricity given the data obtained by Brahe's observations. To this purpose he had to draw on observations made at times when Mars was positioned opposite the Sun, i.e. when Sun, Earth, and Mars were aligned on a straight line, for under these conditions, observing Mars, the sphere of the fixed stars appears the same whether seen from the moving Earth or from the unmoving Sun. Ten such observations of oppositions made by Brahe during the time from 1580 to 1600 existed, and in 1602 and 1604 Kepler recorded two further observations of his own.

In their calculations, Brahe and Longomontanus had, as always, supposed the equant point symmetrical to the Sun with regard to the center of the circle. Therefore three observations were sufficient in order to determine the circular orbit. When choosing the three observations differently, the calculations of Brahe and Longomontanus resulted in different orbits. Kepler, therefore, in his investigations left the equant point open, thus gaining an additional degree of freedom and requiring four observations in order to determine the orbit and the equant point. The calculations became extremely complicated, since the solution could only be obtained by a method of approximation. Having determined the orbit according to four selected oppositions, Kepler then calculated the respective positions of Mars for the remaining eight oppositions, thus obtaining points spread out over the whole Martian orbit. The calculated points along the orbit coincided with the observations with an exactitude of 2 minutes. This deviation was in accordance with Brahe's exactitude of observation.

Thus, the problem might have been considered as having been solved, but Kepler sought to corroborate his result by way of calculating the eccentricities of Mars' orbit directly by drawing on a number of suitable observations. The result of this test, however, was a deviation of 8 minutes between the position of the planet as calculated and as observed. This was more than the exactitude of Brahe's observations allowed for. Kepler concluded that the theory must be faulty and that he would have to try further in order to attempt the greatest possible agreement with the observations. Later, he declared: "These eight minutes led toward the renewal of the whole astronomy." To Longomontanus he wrote at the beginning of 1605*: "Could I have left a doubt about eight minutes, I need never have undergone the endless troubles during the entire year 1604."

Kepler envisaged two possibilities which might have caused the mistake. Copernicus and Brahe had referred in their calculations not to the Sun but to the center of the Earth's orbit. To this Kepler objected, since he was convinced that the moving force originated from the Sun. Accordingly, he decided to transpose all calculations to the Sun as center.

Secondly, Kepler could not see why in the case of the Earth, contrary to all other planets, an equant point should be unnecessary. He therefore thought it advisable, first of all, to attempt to gain a more precise knowledge of the Earth's orbit. He achieved this through a rather ingenious trick, by way of observing the Earth at different times from one and the same point on Mars' orbit, a procedure which was possible since the sidereal period of Mars was well known. The calculations showed that the Earth's motion on its supposedly circular orbit was a non-uniform one, thus disproving Copernicus' assumption.

Furthermore, Kepler recognized an interdependence between the speeds and distances in perihelion and aphelion: The speeds are inversely proportional to the distances Sun-Earth: $v_{Pe}/v_{Ap} = r_{Ap}/r_{Pe}$. By induction Kepler concluded that this theorem must hold true for any planet and for any point of orbit. This, or course, had to be verified.

* (KGW, 15 (1951), 143).

How Kepler finally arrived at the area law which says that the radius vector Sun-planet sweeps over equal areas in equal times, and which became then known as the Second Keplerian Law, need not be outlined in detail. Kepler's derivation has been repeatedly analyzed up to the most recent time.*

To prove his theorem Kepler drew on three different hypotheses which were not compatible. Therefore the Berlin astronomer, Encke (1791-1865) in one of his lectures 1836/37, reproached him with having made two mistakes, neutralizing one another, a reproach, which was later taken up by others, but which is, as Caspar (1928) has shown, was not justified.[3]

By means of the above-mentioned calculations Kepler succeeded in determining the distances Sun-Mars. Returning to his investigations of Mars' orbit, he could easily demonstrate that this orbit was not a circular one. Somewhat prematurely he believed to have found a physical mechanism underlying Mars' motion around the Sun according to which the orbit was an oval symmetrical along its longitudinal axis but not along its lateral axis, in other words, egg-shaped with the blunt edge in the aphelion and the sharp edge in the perihelion. As it turned out, however, this oval was too narrow; toward the vertex of the curve the distances Mars-Sun were smaller than they ought to have been according to the observations. Further calculations led to a bulging curve (via buccosa) equally symmetrical along the lateral axis, which, however, also had to be discarded, until Kepler finally came to realize that an ellipse was the correct reproduction of the distances. In this way it was empirically proved that the planetary orbits are ellipses with the Sun being situated in one focus. The First Keplerian Law was discovered. At the same time, the geometrical qualities of the ellipse as already known to Apollonius showed the validity of the area law. Although it holds true in the perigee and the apogee that the speeds are inversely proportional to the distances, this does not apply to the remaining points of the orbit.

The discovery of these two laws was the outcome of years of laborious research. Already in 1601 Kepler had been certain that the eccentricities of the planetary orbits must be calculated with reference to the real Sun. In the same year he obtained his knowledge about the Earth's orbit and came to the conclusion that the speed of a planet is inversely proportional to its distance from the Sun. This was indicated in a letter to Mästlin dating from December 1601.† Soon afterwards, the precise date not being ascertainable, he must have discovered the area law; to establish the precise date of this discovery has proved impossible. The investigations of Mars' orbit then had to be discontinued, as Kepler applied himself to completing

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* (cf. in particular the comments of the distinguished editor and translator into German of Kepler's work, Max Caspar (1880-1956), in the German edition of the "Astronomia nova", pp.36*-62*, see also Casper[3] (1928), Koyré[15] (1961), Wilson[21,22] (1968, 1972), Aiton (1969, 1978), Krafft[16] (1973), and Russo[18] (1973).

† (KGW, 14 (1949), 202-208).

his work on optics. The entire year of 1604 was taken up by the time consuming calculations in connection with the oval hypothesis. In the spring or summer of 1605 he came to realize that the planetary orbits are ellipses. This can be assumed from a letter Kepler wrote to David Fabricius (1564-1617), a Frisian reverend and astronomer known as discoverer of a variable fixed star, 11 October 1605.*

The publication of the two planetary laws did not take place till 1609 in the book entitled: *Astronomia nova . . . seu Physica coelestis, tradita commentariis de motibus stellae Martis, ex observationibus G. V. Tychonis Brahe* ("New Astronomy, Based on Causes, or Celestial Physics Illustrated by Comments on the Motion of the Planet Mars According to the Observations of the Nobleman Tycho Brahe"). The title indicated that the author's intention was to reveal the causes for the planetary motions. The delays in the printing of this book occurred because of difficulties raised by Brahe's heirs as well as because of a lack of funds.

HARMONICE MUNDI

The Third Keplerian Law is contained in Kepler's third major work entitled *Harmonices mundi libri V* ("Five Books on the Harmony of the World")[10] and published in 1619. It is mentioned there in the third chapter of the fifth book, which is called "The main theorems of astronomy necessary for the investigation of the celestial harmonies", and which is divided into 13 parts. In part 8 it says: "For it is certain and true beyond doubt that the proportions between the orbital periods of any two planets equal exactly one and a half times the proportions of their mean distances from the Sun."† I.e. between the periods T_1, T_2 and the mean distances a_1, a_2 there is the relation $T_1/T_2 = (a_1/a_2)^{3/2}$.

Kepler arrived at this theorem empirically by juggling around the available data on planetary orbits. Twenty-two years earlier he had first raised the question of the connection between period and distance in his *Mysterium cosmographicum*. Now he reports in the above-mentioned sub-chapter of the *Harmonice mundi* that on 8 May 1618 he had thought to have found the looked-for connection, but that he had dismissed it when trying to prove it numerically, until on 15 May he finally became convinced of its validity. His enthusiasm becomes evident in the preface to the fifth book of the *Harmonice mundi* where he says:

> "Eighteen months ago the first dim light appeared at the horizon; three months ago dawn broke; a few days ago the Sun rose in all its glory, and now nothing will keep me back; I am ready to abandon myself to a state of holy ecstasy."‡

In Kepler's own view, the discovery of the theorem that came to be known as the Third Keplerian Law marked but a small, if important, step in his investigations, since his actual aim was the disclosure of the harmonies underlying the concept of

* (KGW, 15 (1951), 240-280).

† (KGW, 6 (1940), 302).

‡ (Cf. Haase (1971)[4]).

creation. Thus, he speculated widely about the acoustic regularities of the planetary orbits and developed a system according to which a connection existed between the speeds in the aphelion and perihelion correspond to the sounds in the major and minor scales. In his words, the celestial motions are but a continuous music arranged for several voices, not to be heard acoustically but to be perceived rationally. These ideas seem rather strange to the exact scientist of today.

APPRECIATION

In retrospect, it becomes evident that the discovery of the laws of planetary motions by Johannes Kepler was brought about by a coinciding of favorable if not indispensable circumstances.

Most important, here was a man who conceived of investigating the logic of the planetary system and reducing the planetary motions to their physical causes. The linking of astronomy and physics was requisite, and a stroke of genius.

A second essential long-term prerequisite was that Kepler was able to avail himself of the data gained by Brahe's long-term observations of the planet Mars. As these data provided the only possible basis for a successful investigation of the planetary orbits, Kepler tended to regard it as divine providence that the religious controversies in Graz had led him to Prague just at the time when Longomontanus had failed in his attempt to calculate the Martian orbit. In order to rightly appreciate his achievement, one must, however, bear in mind that even in the case of the Martian ellipse the minor axis is only one and a half percent smaller than the major axis so that, at first view, it is almost impossible to distinguish this orbit from a circle.

The evaluation of the observational data demanded a good foundation in mathematics as well as skill and perseverance in the execution of the numerical calculations. The difficulties which presented themselves arose from the fact that the mathematical means required for the solution of some of the problems did not yet exist and that even the mathematical notation had not yet been sufficiently developed. Derivations and results had to be circumscribed at great lengths, and the complicated computations had to be carried out without the help of logarithms. Kepler showed imagination in developing mathematical methods, intuition in choosing the observational data just right for his calculations, and indefatigable patience in carrying out the calculations. He invented suitable methods of approximation and did not hesitate to undertake, if necessary, 70 iterations.

Kepler's attempts at arriving at a more precise knowledge of the planetary orbits would not have succeeded if it had not been for the then unique exactitude of Brahe's observations of 2 angular minutes. Prior observations showed at the very best an exactness of 10 angular minutes. If Kepler had had to rely on such data, the deviation of 8 angular minutes in his calculations would not have been sufficient in order to prove the deviation of the Martian orbit from a circle. On the other hand, it would have been equally disadvantageous had Brahe's observations shown the by far greater exactitude which may be obtained today or even only the exactitude which could be obtained in the 18th Century, for then disturbances of the planetary orbits would have become noticeable so that the calculation of the orbits would not have resulted in exact ellipses. With regard to Kepler's calculations these disturbances were of no account.*

Today we know that the Keplerian laws of the two-body problem only apply rigorously if the mass of the planet is negligible in comparison with the mass of the central body. Otherwise the Third Keplerian Law would have to allow for a correction factor depending on the masses m_1 and m_2 of the two planets and the mass M of the central body, reading as follows:

Kepler did not require this correction factor for the planets of the solar system. What then was the reception of Kepler's investigations of the planetary motions? Kepler met with the same response as any famous man propagating an entirely new idea in the sciences. The validity of a new idea takes time to gain credence. Contemporaries are mostly opposed to or reluctant to adopt unusual ways of thinking, and show a considerable degree of scepticism. Accordingly, those whose judgment Kepler valued most - Galilei, Mästlin, Fabricius - did not appreciate his work.

Kepler had hoped for a comment on his *Mysterium cosmographicum* by Galilei and had written to him on 13 October 1597: "Believe me, I would rather meet with the most severe criticism of a single man of understanding then with the applause of the unthinking mass."† But Galilei did not react to Kepler's letter. Only in 1610, after the publication of the "Conversation with the Sidereal Messenger" it came to an exchange of letters between Kepler and Galilei. It is, however, impossible to say how intensively Galilei studied the *Astronomia nova*. In his *Dialogo sopra i due massimi sistemi del mondo* ("Dialogue about the Two Great Systems of the World") (1632), in any case, Galilei still entertained the theory that the motion of the planets was a uniformly circular one - 23 years after the appearance of the *Astronomia nova* establishing the ellipticity of Mars' orbit!

Mästlin had introduced Kepler to the teachings of Copernicus, but never had subscribed to them himself. Even in in 1624 in the 7th edition of his *Epitome*

* (cf. Schmeidler (1971))[19].

† (KGW, 13 (1945), 144-146).

astronomiae he did not propagate the Copernican system. He had no sympathy for Kepler's physical concepts. When Kepler approached his former teacher with his conviction that astronomy and physics were related to each other so closely that one without the other could never achieve perfection, Mästlin warned him against linking the two since only geometry and arithmetic could lead to astronomical results.

Kepler always kept his teacher informed about his scientific plans and the conclusions he had reached. In a letter dating from December 1601,* for instance, he reports in great detail on the progress he had made with regard to the theory of planetary motion. For years, however, Mästlin did not answer Kepler's letters. And when he wrote to him at the beginning of 1605† he found no other excuse for this protracted silence than that he had not known what to reply to a mathematician of such excellence.

Fabricius did show a major interest in the development of Kepler's calculations of Mars' orbit, but when confronted with the proof of the orbit's ellipticity, he did not like the result: He would have preferred a circular orbit (Fabricius' letters to Kepler of 11 January 1606‡ and 20 January 1607.**

Kepler's work did not come to be forgotten, but when astronomers did study it during the first decades after his death, it was rather an exception than the rule if they adopted his laws of planetary motion.†† This may partly be due to the fact that within Kepler's works these laws appear in a context of more general considerations which could not be consented to. It was only Isaac Newton (1643-1727) who helped Kepler to full recognition, deriving from his laws the law of universal gravitation. Kepler himself had not been able to proceed to this very end. Although he came very close to the concept of gravitation, he did not fully appreciate it since he did not conceive the force issuing from the Sun as three-dimensional, but considered it as confined to the planetary planes.

For us, nowadays, Kepler's importance consists mainly - as that of Galilei - in his approach to physical ideas as one of the pathfinders of the exact natural sciences. By establishing the laws of planetary motion he is the founder of celestial mechanics. At the same time, his laws form the basis for space mechanics.

As far as Kepler himself is concerned, the laws which came to bear his name did not represent the main result of his investigations, in spite of the efforts preced-

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* (KGW, 14 (1949), 202-208).

† (KGW, 15 (1951), 131-134).

‡ (KGW 15, (1951), 303-306).

** (KGW, 15 (1951), 376-386).

†† (cf. Russell (1964)[17], Thoren (1974))[20].

ing their discovery and the delight he felt upon his findings. In Kepler's opinion they marked only one step in the investigation of the harmony of the world whose existence he did not doubt. That Kepler was convinced of the significance of his work, however, is indicated in the passus in the preface of the fifth book of the *Harmonice mundi* which summarizes his ideas, and where he says:

> "I write this book for the present or the future. To me this does not matter. And may it take a hundred years till this book finds its reader; God had to wait for six thousand years until his creation was appreciated."

A final assessment of his life and his work finds its astronautical expression in the epitaph which Kepler composed for himself and which survived, although the protestant graveyard outside Regensburg, where Kepler was buried, was destroyed three years after his death when the Swedes besieged the town:

> "*Mensus eram coelos, nunc terrae metior umbras.*
> *Mens coelestis erat, corporis umbra jacet.*
>
> I measured the heavens, and now I shall measure the Earth's shadows.
> Heavenly was my mind, here lie my Earthly remains."

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