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STABILITY AND CONTROL OF A SATELLITE ROCKET

RA-15025

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PROJECT RAND

(AAF PROJECT MX-791)

STABILITY AND CONTROL OF A SATELLITE ROCKET

R. H. FRICK

RA-15025

February 1, 1947

DOUGLAS AIRCRAFT COMPANY, INC.

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*This initial external distribution list includes the distribution of all related technical reports on the satellite vehicle.

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SUMMARY

This report presents the theory and certain design information for the control system of a satellite rocket. The requirements of the control system are, that it must maintain the rocket on a predetermined trajectory in such a way that the rocket, at the end of burning, is flying in a horizontal direction and at an altitude and velocity consistent with a circular or nearly circular orbit.

The proposed control system described in the report consists of a gyro detection device which determines the error in heading of the vehicle from the predetermined heading program. The error signal determines the amount of correcting moment necessary to reduce the error to zero. The correcting moment can be applied by deflecting a system of four auxiliary rocket motors in such a way as to apply the necessary moments in pitch, yaw, and roll to the vehicle. The system represents a closed loop servomechanism and its analysis is carried out in detail with regard to overall stability of the system, transient response characteristics, and determination of system parameters.

In addition to the control system necessary during the initial trajectory, a control system is also proposed which maintains the attitude in the orbit so that a particular side of the vehicle is always presented to the earth. This system depends upon the detection of the instantaneous direction of motion as a pitch and yaw reference, and the detection of the direction of the earth's magnetic field as a roll reference. The error signals can be converted to changes in angular velocity of a flywheel system, which, in turn, changes the angular velocity of the vehicle by conservation of angular momentum in such a way as to correct the heading. The analysis of the orbital control system is far more complicated than the trajectory control system and is presented only schematically at this time pending further development.

LIST OF SYMBOLS

J_1 = moment of inertia of vehicle
f_1 = viscous damping moment coefficient
k = aerodynamic restoring moment coefficient
θ_o = instantaneous angle of vehicle axis
θ_i = programmed angle of vehicle axis
θ = error in angle of vehicle axis
α = angle of attack of vehicle
M_c = control moment
K_o = error control constant for vehicle
K_1 = derivative control constant for vehicle
K_{-1} = integral control constant for vehicle
ω_1 = undamped natural frequency of vehicle
γ_1 = non-dimensional frictional damping constant of vehicle
ζ_1 = non-dimensional damping constant of vehicle
S_1 = non-dimensional integral control constant of vehicle
τ_1 = response time of vehicle
ϕ = maximum angular error of vehicle
T = thrust of one control motor
r = distance from C G to control motor
δ_o = angular deflection of control motor
δ_i = desired angular deflection of control motor
δ = error in angular position of control motor
J_2 = moment of inertia of control motor
f_2 = viscous damping moment coefficient of control motor
k_o = error control constant of control motor
k_1 = derivative control constant of control motor
k_{-1} = integral control constant of control motor
ω_2 = undamped natural frequency of control motor
ζ_2 = non-dimensional damping constant of control motor
γ_2 = non-dimensional frictional damping constant of control motor
S_2 = non-dimensional integral control constant of control motor
τ_2 = time constant of control motor
δ_m = maximum error of control motor position
M_m = maximum control moment required
M_P = pitch control moment
M_Y = yaw control moment
M_R = roll control moment

LIST OF SYMBOLS *(Cont'd)*

r_p = moment arm in pitch and yaw
r_R = moment arm in roll
δ_p = effective control motor deflection in pitch (2 motors)
δ_Y = effective control motor deflection in yaw (2 motors)
δ_R = effective control motor deflection in roll (4 motors)
δ_{in} = desired deflection of the nth control motor (n = 1-4)
δ_{on} = actual deflection of the nth control motor (n = 1-4)
θ_i = desired pitch angle of vehicle
ψ_i = desired yaw angle of vehicle
ϕ_i = desired roll angle of vehicle
θ_o = actual pitch angle of vehicle
ψ_o = actual yaw angle of vehicle
ϕ_o = actual roll angle of vehicle
θ = error in pitch angle
ψ = error in yaw angle
ϕ = error in roll angle
δ_n = error in angle of nth control motor (n = 1-4)
s_n = moment signal to nth control motor (n = 1-4)
M_n = correcting moment applied to vehicle by nth control motor
M = mass of the satellite rocket
y = displacement normal to flight path
T_o = thrust of main jet
J'' = moment of inertia of second stage
V'' = volume of second stage
q = dynamic pressure
M_a = aerodynamic moment
α_1 = angle of attack at end of stage I
$\overline{\omega}_1$ = vector angular velocity of vehicle in orbit
$\overline{\omega}_F$ = vector angular velocity of flywheels
J_F = moment of inertia of flywheels
P = chamber pressure for molecular beam detector
P_o = atmospheric pressure at orbital altitude
v = orbital velocity
ρ_o = atmospheric density at orbital altitude
R_o = universal gas constant
M' = molecular weight of atmosphere at orbital altitude
T = absolute temperature at orbital altitude

LIST OF SYMBOLS *(Cont'd)*

τ_p = response time of molecular beam detector
E = voltage output of molecular beam detector

LIST OF SYMBOLS

(APPENDIX I)

θ_i = input angle
θ_o = output angle
θ = error angle
M_c = control moment on output member
M_o = external moment on output member
J = moment of inertia of moving system
f = viscous friction moment coefficient
K_o = error control constant
K_1 = derivative control constant
K_{-1} = integral control constant
ω_n = undamped natural frequency of system
γ = non-dimensional frictional damping constant
ζ = non-dimensional damping constant
S = non-dimensional integral control constant
α_o = steady state deflection of system due to M_o if no integral control is present
$\overline{\theta}_i$ = Laplace transform of θ_i
$\overline{\theta}$ = Laplace transform of θ
M_m = maximum required control moment
δ = non-dimensional derivative control constant

STABILITY AND CONTROL OF A SATELLITE ROCKET

INTRODUCTION

The satellite rocket as now proposed is to operate in the following manner. The rocket starts from rest at ground level and ascends vertically for about one-tenth of the burning period of the first stage with zero angle of attack. At one-tenth of the burning time an angle of attack program is initiated which reaches a maximum at about one quarter of the first burning period. After this it decreases slowly to zero at nine-tenths of the burning period and remains zero for the rest of the trajectory.

The angle of attack program determines the shape of the trajectory, and from nine-tenths of the first burning period on, the curvature of the path is affected only by gravity and the residual aerodynamic forces. At the end of the first burning period of about 110 seconds, the first stage is dropped and for an interval of 1.2 seconds no thrust is applied. Then the second stage motor is started and operates throughout the second burning period of about 110 seconds. At the end of this time there is another 1.2 second separation period, during which the second stage is dropped. The third stage motor is then started and burns for approximately 100 seconds. Then the motor is shut off and the rocket is allowed to coast for a period of about seven minutes, the exact length of coasting being determined by the amount of error in position and velocity existing at the start of coasting. At the end of coasting the third stage motor is operated for an interval of about 10 seconds, which should be sufficient to put the rocket on its orbit. From here on the rocket becomes a satellite and no further thrust is necessary.

The control system of the vehicle must be such that the motions of the body about its center of gravity are stable at all times during the trajectory. The system must also be capable of exerting sufficient moment on the vehicle to maintain it in the desired heading against the action of existing aerodynamic moments and possible moments due to misalignment of the rocket motor. Finally, the system should be able to exert sufficient moment to correct an error in the vehicle attitude within a specified response time.

After the vehicle is in its orbit, the control system must be able to keep the heading tangent to the orbit and the roll velocity zero.

In this way, the same side of the vehicle would be presented to the earth and the amount of power necessary for the vehicle borne radar could be materially reduced by use of directional antennae.

The vehicle in flight should follow a programmed trajectory. This program can be obtained by calculating from the trajectory data[1] the time variation of the vehicle heading with reference to the initial vertical.

For references see page 28

Any deviations of the actual heading from this desired heading would result in error signals, which cause correcting moments to be applied to the vehicle to return it to the specified program.

These correcting moments can be applied by means of four auxiliary control rockets mounted symmetrically about the main rocket motor. The equilibrium position of these control rockets is parallel to the axis of the main jet; one pair being capable of motion in the pitch plane, while the other pair would move in the yaw plane. Thus, by consistent deflections of one pair, moments in pitch are produced, while consistent deflections of the other pair create a yaw moment. On the other hand, opposing deflections of each pair give a roll moment. By suitable deflections of the four rockets, any desired combination of pitch, yaw, and roll control moments can be applied to the vehicle.

After the vehicle is in its orbit, the attitude control is based on the direction of the flight path for pitch and yaw and the direction of the earth's magnetic field for roll. The actual application of correction moments can be achieved by means of a system of three flywheels with mutually perpendicular axes. The angular velocities of these flywheels are adjusted so that, in accordance with the conservation of angular momentum, the vehicle makes one rotation about its center of gravity in the plane of the orbit per orbital period, and at the same time maintains zero yaw and roll angles.

A more complete description of the control system is given in the following sections.

THEORY

The problem of stability and control of the satellite rocket divides itself into two parts. The first of these involves the stability and control of the rocket during the powered trajectory and the coasting period. During this time the rocket is maintained on a programmed path, such that at the end of the powered flight, the rocket is on a satisfactory approximation to a circular orbit. The second part of the problem, involving the stability and control of the rocket after it is in the orbit, is a matter of attitude control, since the actual orbital path has been determined by the speed and direction of motion of the rocket at the end of the powered flight. It is desirable that the control system in this part of the orbit be such that a particular side of the rocket always faces the earth. These two control systems are discussed in more detail in the following paragraphs.

A. Trajectory Control

In the case of the German V-2 Rocket, the control of the trajectory was achieved by means of jet vanes placed in the main rocket jet, in such a way that a portion of this jet could be deflected to give the necessary control moments about the center of gravity of the vehicle. The method has three main disadvantages. In the first place, the jet vanes are exposed to the extremely high temperatures of the jet and as a result are progressively burned away during the flight. Since the burning time for the satellite rocket is about twice that for the V-2, it would be necessary to develop some new heat resistant vane material to give satisfactory control over the whole trajectory. In the second place, the presence of jet vanes in the main jet introduces an additional drag, which reduces the effective thrust of the motor and thus increases the fuel requirement. Finally, it is understood that the Germans had to supplement the jet vanes with air rudders in order to obtain adequate roll control of the V-2, and as a result were of the opinion that some other control mechanism would be more satisfactory.

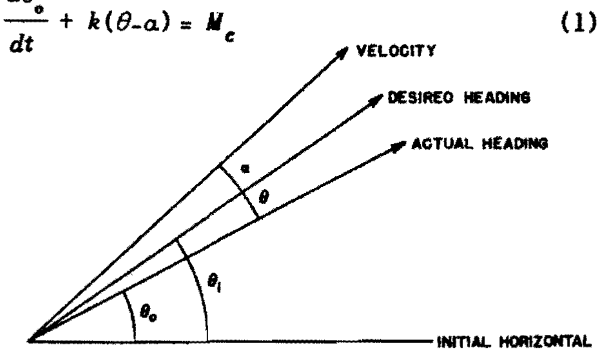
In view of these disadvantages, it was decided that the control of the satellite rocket should be accomplished by means of four rocket motors mounted symmetrically about the axis of the vehicle. In their equilibrium position, these four motors are directed parallel to the axis of the vehicle. One pair of the motors is capable of rotation in the pitch plane, while the other pair rotates in the yaw plane. With a system of this type, correcting moments in pitch and yaw can be applied to the vehicle by equal deflections of the appropriate pair of control rockets. In this case the resulting moment is the sum of the pitch or yaw moments of the two rockets of a particular pair. In the case of roll control, the two rockets are deflected equal and opposite amounts so that the resulting roll moment is the sum of the roll moments of the two rocket motors. Roll control can thus be supplied by all four of the control rockets.

A control system of the above type overcomes the disadvantages inherent in the jet vane method. In the first place, there will be no variation in the amount of control due to burning away of the control members. Secondly, there is no additional drag introduced by the presence of the control rockets, and since for small deflections, the main component of their thrust is in the direction of the vehicle axis, this thrust contributes to the propulsion of the vehicle. Finally, the control motors can be designed so as to give any required roll moment.

In order to determine the magnitudes of the thrusts necessary in the control rockets, and also the parameters of the associated servo system, the following equations of motion of the vehicle are set up.

$$J_1 \frac{d^2\theta_o}{dt^2} + f_1 \frac{d\theta_o}{dt} + k(\theta - \alpha) = M_c \qquad (1)$$

The angles in this equation are shown in Fig. (1).



ANGULAR REFERENCE FOR CONTROL SYSTEM
FIG I

The control moment supplied by the auxiliary rocket motors is determined on the basis of the servo analysis in Appendix I. From this analysis, it seems that a suitable control moment would be given by the relation

$$M_c = K_o\theta + K_1\frac{d\theta}{dt} + K_{-1}\int \theta dt \quad . \tag{2}$$

By eliminating M_c between Eqs. (1) and (2) the complete control equation becomes

$$J_1\frac{d^2\theta_o}{dt^2} + f_1\frac{d\theta_o}{dt} = (K_o + k)\theta + K_1\frac{d\theta}{dt} + K_{-1}\int_o^t \theta dt + k\alpha \quad . \tag{3}$$

By substituting in Eq. (3) the relation

$$\theta_o = \theta_i - \theta \tag{4}$$

the control equation takes the form

$$J_1\frac{d^2\theta_i}{dt^2} + f_1\frac{d\theta_i}{dt} = J_1\frac{d^2\theta}{dt^2} + (f_1 + K_1)\frac{d\theta}{dt} + (K_o + k)\theta + K_{-1}\int_o^t \theta dt + k\alpha \quad . \tag{5}$$

A comparison of Eq. (5) with Eq. (51) of Appendix I shows that the form of the two equations is identical, with the exception that the aerodynamic restoring moment coefficient is added to the error control constant. In the above equation the term $k\alpha$ represents an external moment, which must be counteracted by the control system in order to hold the vehicle in the desired tilt program.

As in Appendix I the quantities ω_1, γ_1, ζ_1, S_1, and τ_1 can be introduced by the relations

$$\begin{aligned}
\omega_1^2 &= \frac{K_o + k}{J_1} \\
\gamma_1 &= \frac{f_1}{2\sqrt{(K_o + k)J_1}} \\
\zeta_1 &= \frac{K_1 + f_1}{2\sqrt{(K_o + k)J_1}} \\
S_1 &= \frac{K_{-1}}{K_o + k}\sqrt{\frac{J_1}{K_o + k}} \\
\tau_1 &= \frac{2(K_o + k)}{K_{-1}} \quad .
\end{aligned} \tag{6}$$

These new variables correspond to those introduced in Appendix I without the subscripts. However, in this application of the theory, the quantities J_1, k, and f_1 are functions of the position along the trajectory. Thus, it is not possible to specify a fixed value for any one of these parameters. Instead, the procedure which seems most reasonable is to solve the above equation for the desired control constants as follows:

$$K_o = \frac{4J_1}{S_1^2 \tau_1^2} - k$$

$$K_1 = \frac{4J_1 \zeta_1}{S_1 \tau_1} - f_1 \qquad (7)$$

$$K_{-1} = \frac{8J_1}{S_1^2 \tau_1^2} \quad .$$

From these equations, it is possible to determine the average values of K_o, K_1, and K_{-1} by substituting average values of J_1, k, and f_1 for a particular stage, the desired values of τ_1 and the ideal values of ζ_1 and S_1 as determined in Appendix I.

$$\zeta_1 = \frac{\sqrt{3}}{2}$$

$$S_1 = \frac{\sqrt{3}}{9} \quad .$$

With the values of K_o, K_1, and K_{-1}, so determined, the actual values of ω_1, ζ_1, S_1, and τ_1 can be calculated for any position along the flight path.

The stability of the system can be checked by plotting $2\zeta_1$ and S_1 on the same graph as a function of time. Then, in accordance with the stability condition, Eq. (69) of Appendix I, as long as the curve of $2\zeta_1$ lies above that for S_1, the system is stable.

A plot of the response time τ_1, as a function of time, indicates the time necessary for a given error to be reduced to about one-tenth of its initial value.

The application of the above method to the determination of the K_o, K_1, and K_{-1}, should be regarded as a first approximation. It may be necessary to adjust these values somewhat in order that the range of variation of ζ_1 over the trajectory should be between .6 and 1.0, and that the value of S should not exceed .3. Such adjustment may be particularly necessary in the second stage, which at present is aerodynamically unstable in the first few seconds. Sufficient control must be available to counteract the initial instability, even though this results in too much control in the latter part of the stage, when aerodynamic forces are negligible.

The specification of the size of the control motors can be obtained from Eq. (61a) of Appendix I, which gives the maximum required control moment for the correction of an error angle ϕ as follows:

$$M_c = \frac{108 J_1 \phi}{\tau_1^2} \quad . \tag{8}$$

It appears probable that the maximum M_c would occur at the beginning of the stage when J_1 is a maximum, since τ_1 is essentially constant over a given stage. Thus, the control motor must be large enough to supply a moment M_c at maximum deflection δ_o by the relation

$$M_c = 2Tr \sin \delta_o = 2Tr\delta_o \quad . \tag{9}$$

Equating the two expressions for M_c and solving for the thrust T, the following result is obtained:

$$T = \frac{54 J_1 \phi}{\tau_1^2 r \delta_o} \quad . \tag{10}$$

In the following table the sizes of the control motors necessary on the various stages are listed as calculated from Eq. (10). It is assumed that

$$\delta_o = 15°$$

$$\phi = 1°$$

$$\tau_1 = 4 \text{ sec.}$$

Table I

Stage No.	J_1 (slug ft^2)	r (ft)	T (lbs)	T(V-2) (lbs)
1	1.385×10^6	18.00	17,300	22,800
2	5.68×10^4	13.33	959	1,260
3	8.65×10^2	8.37	23.2	30.6

In the last column of Table I are listed the thrusts necessary, as determined by calculating the control moment available in the V-2, by converting this into an equivalent rocket control motor and by scaling this motor up to the size of the satellite rocket. In making this scale-up, the moment of inertia of the first stage of the satellite rocket was taken as three times that of the V-2 and the moment arm for the V-2 was taken as 18 ft.

A comparison of the last two columns of Table I indicates that the V-2 scale-up is somewhat larger. In view of the result, it seems reasonable to adopt the following values for the control rocket thrust.

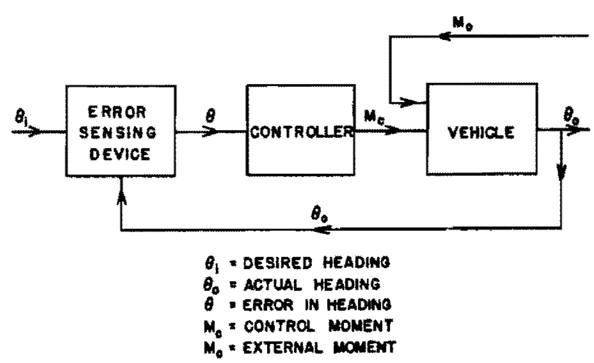
Stage 1	22,000 lbs
Stage 2	1,200 lbs
Stage 3	30 lbs

While these values are larger than predicted by theory, it is then possible to attain a somewhat smaller time constant or a somewhat larger range in the error angle φ. Actually, the thrust values obtained from Eq. (10) represent a minimum value of the thrust necessary. Any increase in this value above the minimum, increases the control moment available, and at the same time increases the sensitivity of the positioning servo for the rocket motors. The upper limit of the control rocket thrust is one-fourth of the total thrust for a given stage, and the actual thrust selected must be such as to give as much control moment as possible without increasing the precision of the positioning servo unduly.

The servo systems associated with the pitch and yaw control of the vehicle should be identical, since the moments of inertia and the lever arms about the center of gravity are the same for both degrees of freedom. The only difference in these systems is that there is an angle of attack program in the pitch plane, and the control system must exert sufficient control moment to hold the vehicle at the prescribed angle of attack. However, with the thrusts specified, it should be entirely possible to follow the prescribed angle of attack program against the existing aerodynamic moment $k\alpha$ of Eq. (5).

The servo system for roll control has certain simplifications over those for pitch and yaw, since in roll the aerodynamic restoring moment coefficient k is zero and the aerodynamic damping moment coefficient is also small. Thus, the only variable quantity in the determination of K_o, K_1, and K_{-1} in Eqs. (7) is the moment of inertia.

The analysis of the overall servo system given above considers a servo loop as shown in Fig. 2. In this figure, the programmed angular position of the vehicle axis θ_i is fed into an error sensing device together with the actual position of of the vehicle axis θ_o. The error sensing device then produces the error angle θ which is fed into the controller. The controller performs the operation indicated by Eq. (2) and produces a control moment M_c which is applied to the vehicle. The control moment produces a change in the position of the axis θ_o, which is fed back into the error sensing device. The adjustment process continues until a steady state condition is reached in which the values of θ_i and θ_o are equal.

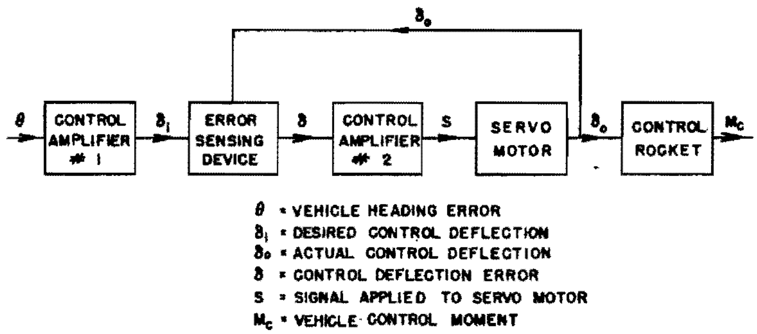


MISSILE SERVO CONTROL LOOP

FIG. 2

The actual servo system is somewhat more complicated than that shown in Fig. 2. In order to convert the error angle θ into a control moment M_c, it is necessary to position the two control rockets to a particular angle δ_i such that

$$M_c = 2Tr\delta_i \quad . \tag{11}$$



AUXILIARY SERVO CONTROL LOOP

FIG. 3

Thus, the controller shown in Fig. 2, involves some sort of a positioning servo system as shown in Fig. 3. In such a system, the error signal θ is fed into Control Amplifier #1, which contains the vehicle control constants in such a way that for an input θ, the output signal is proportional to the desired deflection of the rocket motors as follows:

$$\delta_i = \frac{1}{2Tr}\left(K_o\theta + K_1\frac{d\theta}{dt} + K_{-1}\int \theta dt\right) \quad . \tag{12}$$

The value of δ_i, together with a signal proportional to the actual rocket motor position δ_o, is fed into an error sensing device which produces the error in rocket motor position δ. The error signal is fed into Control Amplifier #2, which has a response such that the resulting output S is proportional to the moment necessary to correct the rocket motor position. This signal applied to the servo motor causes a change in the rocket motor position δ_o which is fed back to the error sensing device. The process continues until the rocket motor position δ_o is equal to the input position δ_i, at which time the desired moment M_c is applied to the vehicle.

It is necessary that the design of the auxiliary servo system be such that its time constant is short compared with the time constant of the vehicle. If this is not true, the resulting time lag between the error signal and the application of the control moment might be sufficient to cause instability in the overall system.

The analysis of the auxiliary servo system is similar to that of the overall system. The equation of motion of the rocket motor being given by

$$J_2 \frac{d^2\delta_o}{dt^2} + f_2 \frac{d\delta_o}{dt} = M_c \quad , \tag{13}$$

where the control moment is determined by the relation

$$M_c = k_o\delta + k_1 \frac{d\delta}{dt} + k_{-1}\int_o^t \delta\, dt \quad . \tag{14}$$

The control moment equation determines the characteristic of the control amplifier #2, since the output of the amplifier must be proportional to M_c for an input δ.

By elimination of M_c between the two Eqs. (13) and (14) the equation of motion becomes

$$J_2 \frac{d^2\delta_o}{dt^2} + f_2 \frac{d\delta_o}{dt} = k_o\delta + k_1 \frac{d\delta}{dt} + k_{-1}\int_o^t \delta dt \quad . \tag{15}$$

Substitution of the relation

$$\delta_o = \delta_i - \delta \tag{16}$$

gives

$$J_2 \frac{d^2\delta_i}{dt^2} + f_2 \frac{d\delta_i}{dt} = J_2 \frac{d^2\delta}{dt^2} + (k_1 + f_2)\frac{d\delta}{dt} + k_o\delta + k_{-1}\int_o^t \delta dt \quad . \tag{17}$$

As before the quantities ω_2, ζ_2, γ_2, S_2, and τ_2, can be defined by the relations

$$\begin{aligned}
\omega_2{}^2 &= \frac{k_o}{J_2} \\
\zeta_2 &= \frac{k_1 + f_2}{2\sqrt{k_o J_2}} \\
\gamma_2 &= \frac{f_2}{2\sqrt{k_o J_2}} \\
S_2 &= \frac{k_{-1}}{k_o}\sqrt{\frac{J_2}{k_o}} \\
\tau_2 &= \frac{2k_o}{k_{-1}} \quad .
\end{aligned} \tag{18}$$

Substitution of these quantities in Eq. (17) reduces it to the form

$$\frac{d^2\delta_i}{dt^2} + 2\gamma_2\omega_2 \frac{d\delta_i}{dt} = \frac{d^2\delta}{dt^2} + 2\zeta_2\omega_2 \frac{d\delta}{dt} + \omega_2{}^2\delta + S_2\omega_2{}^3 \int_0^t \delta dt \quad . \qquad (19)$$

This equation is of the same form as that of Eq. (53), Appendix I. Again it appears to be desirable to have a system with ζ_2 and S_2 equal to $\sqrt{3}/2$ and $\sqrt{3}/9$, respectively.

Thus, the parameters for the auxiliary servo system are determined by the relations

$$\begin{aligned} k_o &= \frac{4J_2}{S_2{}^2\tau_2{}^2} = 108\,\frac{J_2}{\tau_2{}^2} \\ k_1 &= \frac{4\zeta_2 J_2}{S_2\tau_2} - f_2 = 18\,\frac{J_2}{\tau_2} - f_2 \\ k_{-1} &= \frac{8J_2}{S_2{}^2\tau_2{}^3} = \frac{216J_2}{\tau_2{}^3} \quad . \end{aligned} \qquad (20)$$

The determination of the maximum moment to be applied by the servo motor, which moves the control rocket, is again determined by the relation

$$M_m = \frac{108J_2\delta_m}{\tau_2{}^2} \quad , \qquad (21)$$

where the maximum error δ_m of the control motor position is taken as the maximum deflection.

The determination of the parameters of such a servo system is greatly simplified over the case of the overall system, since the quantities J_2 and f_2 are not variable. Thus, the constants k_o, k_1, k_{-1}, and M_m are determined directly from Eqs. (20) and (21) without resorting to any averaging process.

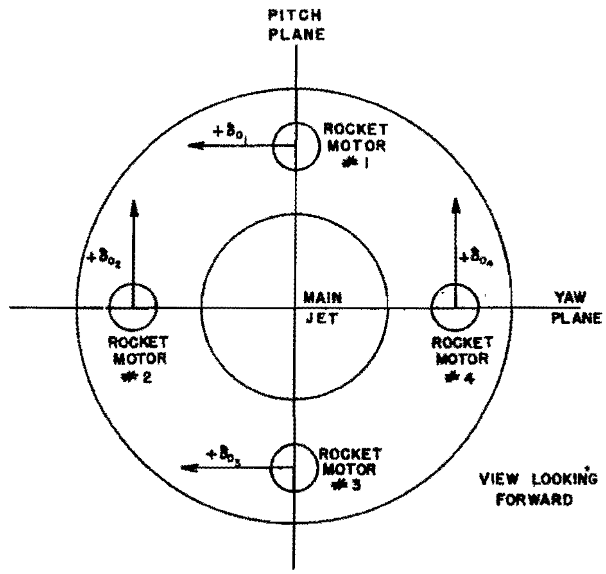
In the discussion thus far, it is assumed that the overall servo system accepts an attitude error signal and converts it into a control motor deflection, which causes a correcting moment to be applied to the vehicle. However, in order to apply control moments in pitch, yaw, and roll, by means of the four control motors, as indicated earlier in this report, it is necessary that the deflections of the four control motors be of different amounts. Thus, the three control moment signals from the first control amplifiers in pitch, yaw, and roll must be combined in different ways to operate each of the control motors.

Suppose that the control motors are located as shown in Fig. 4; the arrows indicate the positive direction of the deflections δ_{i1}, δ_{i2}, δ_{i3}, and δ_{i4} of each motor. If M_P, M_Y and M_R represent the necessary control moments in pitch, yaw, and roll, then the output signals of the first pitch, yaw, and roll amplifiers δ_P, δ_Y, and δ_R, are given by the relations

$$M_P = 2Tr_P\delta_P$$

$$M_Y = 2Tr_P\delta_Y \tag{22}$$

$$M_R = 4Tr_R\delta_R \quad .$$



CONTROL MOTOR ARRANGEMENT
AND DEFLECTION SIGN CONVENTION
FIG. 4

These moment equations are based on the assumption that two motors deflected through an angle of δ_P produce a moment M_P, two motors deflected through an angle δ_Y produce a moment M_Y, and four motors deflected through an angle of δ_R produce a moment M_R. In the actual system these moments are produced simultaneously in accordance with the following relations:

$$M_P = Tr_P(\delta_{i2} + \delta_{i4})$$

$$M_Y = Tr_P(\delta_{i1} + \delta_{i3}) \tag{23}$$

$$M_R = Tr_R(\delta_{i1} - \delta_{i3} + \delta_{i4} - \delta_{i2}) \quad .$$

The above equations do not in themselves give a unique solution for δ_{i1}, δ_{i2}, δ_{i3}, and δ_{i4}, and it is necessary to make the additional assumption that half of the roll moment is produced by each pair of rocket motors, so that

$$\delta_{i1} - \delta_{i3} = \delta_{i4} - \delta_{i2} \quad . \tag{24}$$

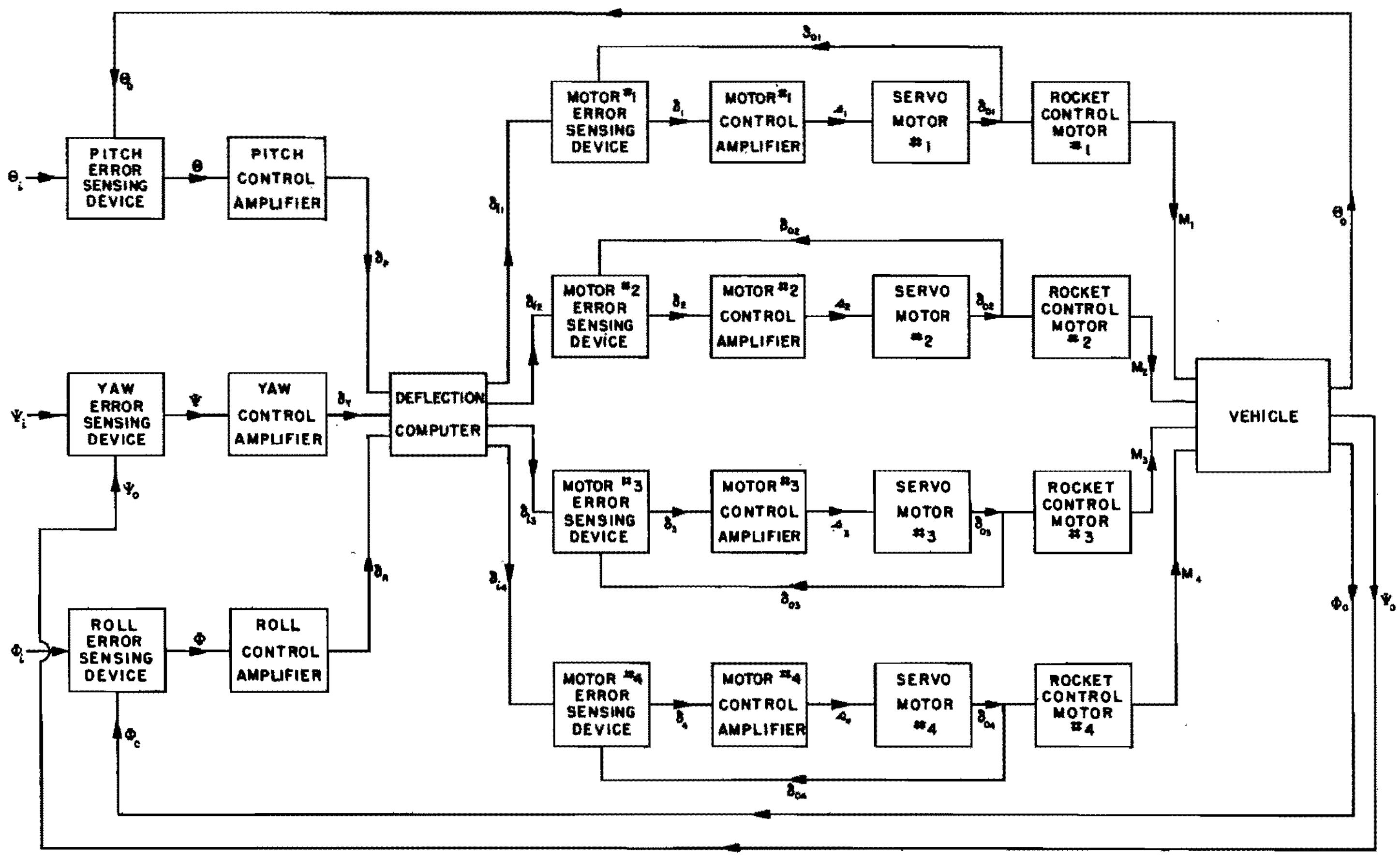
By combining Eqs. (22), (23), and (24) it can be shown that

$$\begin{aligned}
\delta_{i1} &= \delta_Y + \delta_R \\
\delta_{i2} &= \delta_P - \delta_R \\
\delta_{i3} &= \delta_Y - \delta_R \\
\delta_{i4} &= \delta_P + \delta_R \quad .
\end{aligned} \tag{25}$$

Thus, the output of the first control amplifiers in pitch, yaw, and roll are proportional to δ_P, δ_Y, and δ_R. These are combined in accordance with Eq. (25), and the four resulting signals δ_{i1}, δ_{i2}, δ_{i3}, and δ_{i4} are applied to the four servo systems which position the rocket motors. The block diagram of Fig. 5 indicates the arrangement of the complete pitch, yaw, and roll control system.

In the system, the programmed attitude of the vehicle in pitch, yaw, and roll, θ_i, ψ_i, and ϕ_i is fed into the respective error sensing devices, together with the actual attitude of the missile θ_o, ψ_o, and ϕ_o. The error sensing devices produce signals proportional to the errors in pitch, yaw, and roll, θ, ψ, and ϕ. These error signals are accepted by their respective control amplifiers, which produce outputs proportional to the three control moments needed, and likewise, to the deflections of the control motors necessary to produce each of these control moments separately. The deflection signals δ_P, δ_Y, and δ_R are fed into a deflection computer, which operates in accordance with Eqs. (25), and produces four signals δ_{i1}, δ_{i2}, δ_{i3}, and δ_{i4} proportional to the deflections of the four control motors required to produce the desired control moments simultaneously. The position signals are combined with the corresponding actual positions of the control rockets δ_{o1}, δ_{o2}, δ_{o3}, and δ_{o4}, by the error sensing devices to give the position errors of the four control rockets. The error signals are converted by the motor control amplifiers into signals s_1, s_2, s_3, and s_4, which are proportional to the required moments on the four servo motors. These signals cause deflections δ_{o1}, δ_{o2}, δ_{o3}, and δ_{o4} of the four control motors, which are fed back to the four error sensing devices to complete the auxiliary servo loop, so that the four control motors take up the desired positions δ_{i1}, δ_{i2}, δ_{i3}, and δ_{i4}. The four deflections of the control motors exert four vector moments M_1, M_2, M_3, and M_4, on the vehicle. The components of M_2 and M_4 about the pitch axis produce the desired correcting moment M_P, while their components about the roll axis produce a moment $\frac{1}{2}M_R$. Similarly, the components of M_1 and M_3 about the yaw axis produce the moment M_Y, while their components about the roll axis produce an additional moment of $\frac{1}{2}M_R$. The resulting position of the vehicle in pitch, yaw, and roll, θ_o, ψ_o, and ϕ_o, is fed back to the pitch, yaw, and roll error sensing devices so that when the vehicle attains the desired attitude specified by θ_i, ψ_i, and ϕ_i, there is no input to the pitch, yaw, and roll amplifiers, and no further adjustment occurs.

The use of such a control system has certain advantages when used in connection with a programmed trajectory. In the first place, if an external moment is applied to the vehicle, such as the moment resulting from a misalignment of the main jet,



COMPLETE SATELLITE CONTROL SYSTEM
FOR TRAJECTORY

FIG. 5

or the aerodynamic moment due to the tilt program, the system reduces the resulting error to zero as shown in Eq. (58a) of Appendix I. This is accomplished by the integral response of the system, which builds up an equal and opposite control moment, which completely cancels the applied external moment. Such a response is in contrast to the ordinary error or derivative control systems, which require a constant error in order to exert a control moment opposing an external moment. Such a constant attitude error could result in serious distortion of the rocket trajectory.

A second advantage of the above control system is its effect on the lateral motion of the rocket due to the thrust components normal to its path. It can be shown that the equation of motion for such a lateral displacement in the pitch plane is given by

$$M \frac{d^2y}{dt^2} = T_\circ \theta + 2T\delta_p \quad , \tag{26}$$

where the two terms on the right side of the equation represent the lateral forces exerted by a deflection θ of the main jet and a deflection δ_p of the two control rockets.

The lateral velocity can be evaluated by integrating this equation to give

$$\frac{dy}{dt} = \frac{T_\circ}{M} \int_\circ^t \theta dt + \frac{2T}{M} \int_\circ^t \delta_p dt \quad . \tag{27}$$

Since both θ and δ_p vary in accordance with an integral control system, it follows that if t becomes very large, both of the integrals vanish, so that there is no net lateral impulse applied to the system and no resulting lateral velocity due to the correction process. This can be verified by substituting the appropriate time variations of θ and δ_p in the lateral velocity expression.

The lateral displacement of the vehicle can be obtained by a second integration of the equation of motion to give

$$y = \frac{T_\circ}{M} \int_\circ^t \int_\circ^t \theta dt + \frac{2T}{M} \int_\circ^t \int_\circ^t \delta_p dt \quad . \tag{28}$$

In this case it turns out that the second integrals do not vanish, so that after a correction process the rocket is displaced laterally from its programmed path. However, the amount of the displacement is of the order of a few feet, which is negligible for all practical purposes. The lateral displacement could also be eliminated by second integral control term, but in view of the negligible displacement, it hardly seems worthwhile to go to any such refinement.

The control system outlined above assumes that there is no drift in the free gyros, and also that there is no variation in the thrust of the motors. Under these conditions, the input program for the position of the vehicle axis results in the desired tilt program and flight path. However, if there is any drift of the gyros or any fluctuations in thrust, then while the vehicle is held in the correct position with respect to its time program, it is not necessarily in the correct tilt program with reference to the instantaneous flight path. The effect could be particularly serious at two points along the trajectory; first, at the separation of stage 1 and 2, and second, at the end of the coasting period.

In the first case, during the 1.2 second interval of separation, no control moment is available since the control motors of the second stage cannot be started until the first stage is out of the way. As the proposed design of the second stage is aerodynamically unstable, it is possible that the existing q values during separation might cause appreciable attitude errors to develop. This can be seen from the following derivation of the angular motion during separation:

$$J'' \frac{d^2\alpha}{dt^2} = M_\alpha = aqV''\alpha \quad , \tag{29}$$

where "a" is a proportionality constant in the relation between aerodynamic moment and angle of attack. The solution of the equation gives

$$\alpha = \alpha_1 \cosh \left(\frac{aqV''}{J''}\right)^{1/2} t \quad , \tag{30}$$

which shows that the angle of attack at time t is proportional to and greater than the initial angle of attack at the beginning of separation. The actual value of α can be found by substituting

a = .015 (For Mach Number 5.5)

q = 150 lbs/sq ft

V'' = 657 cu ft

J'' = 56,800 slug ft ^{2}

t = 1.2 sec .

These values give the relation for α at the end of the 1.2 second separation time as

$$\alpha = 1.019\ \alpha_0 .$$

Thus, it appears that the change of angle of attack during separation is negligible.

In the second case, at the end of the coasting period, the angle of attack of the rocket should be less than two degrees, in order that the final thrust should put the rocket on its orbit with the required accuracy of ± ½° from the horizontal. With existing gyro systems, the drift of the free gyros can be made less than ½° in the ten minutes elapsed up to the end of coasting. Thus, if the deviations of the trajectory angle from its predicted value, due to erratic thrust, non-standard aerodynamic conditions, and errors in the control system, can be reduced to less than a degree, then the accuracy of 2° in the angle of attack can be achieved. If the tolerances in the flight path cannot be met, it may be necessary to use a molecular beam detector of the type described under Orbital Control Equipment, page 23, in place of the gyro detection system during the latter part of the coasting period. In this way the errors are measured from the actual flight path and the control would be able to reduce the error to zero; thus giving a zero angle of attack.

With regard to the method of control during coasting, it seems that in view of the accuracy required, that the control rockets for the third stage should continue to burn throughout the coasting period in order to supply the desired control moments. (See Appendix II.) It can be shown[*] that the continuous burning of the control rockets can be accomplished without appreciable increase in gross weight or decrease in payload.

B. Orbital Control

In the case of the orbital control system, it is obviously impossible to consider any system which programs the heading of the vehicle as a function of time, since an extremely small error in the time scale of this program results in a cumulative error in the heading of the vehicle in the orbit, and if a directional vehicle-borne radar is used, it would result in long periods during which the vehicle could not communicate with stations on the ground. Thus, any attitude control of the vehicle must be based on measurements made in it at the time the correction is necessary. One reference system which suggests itself is the instantaneous direction of motion of the vehicle and the direction of the earth's magnetic field. If the control system can be arranged so that the longitudinal axis of the vehicle is kept in the direction of the instantaneous velocity of the center of gravity, and a particular diameter of the vehicle kept in the direction of the earth's magnetic field, then the desired attitude control for the orbit is achieved.

Such a control reference requires some sort of detector which determines the direction of motion of the vehicle with reference to a set of axes fixed in the vehicle. From such a determination the instantaneous angles of pitch and yaw of the vehicle are obtained. One method by which this can be done is by means of a molecular beam device, which utilizes the flow of extremely rarefied air into a small chamber in the wall of the vehicle. The resulting pressure in the chamber is a function of the attitude of the vehicle with respect to the air flow, and a combination of several such detectors gives an unambiguous determination of the pitch and yaw angles of the vehicle. The system is discussed more fully in the section on equipment.

In the case of the roll detection device, the deviation of the yaw axis from the earth's magnetic field can be measured by means of some sort of compass device whose axis coincides with the roll axis. Actually, the component of the earth's field, normal to the orbit, is not necessarily perpendicular to the plane of the orbit. However, the magnetic device can be used to set the zero position of a roll gyro at specific points along the trajectory, so that the drift of the roll gyro between zero adjustments is not serious.

If such a reference system is used, all error and error rate signals are measured with reference to a coordinate system moving with the vehicle with its X axis tangent to the trajectory. As the coordinate system has an angular velocity in an inertial frame equal to the orbital angular velocity, the desired control condition is that the vehicle have zero angular velocity in the moving reference system, and likewise that its axis be lined up along the X axis of the moving system. Mathematically, this can be expressed as

$$\dot{\theta} = \dot{\psi} = \dot{\phi} = 0$$

$$\theta = \psi = \phi = 0 \quad .$$

Any deviations from the equilibrium condition are indicated by the detection devices discussed above, and from their readings the instantaneous angular momentum of the vehicle $J_1\bar{\omega}_1$ in the moving reference frame can be determined. The angular momentum can be varied by means of a system of flywheels mounted in the vehicle.

If the angular momentum of the flywheel system is instantaneously $J_F\omega_F$, then the total angular momentum in the moving reference system is given by

$$J_1\bar{\omega}_1 + J_F\bar{\omega}_F = \text{const} \quad .$$

It is desired to reduce $\bar{\omega}_1$ to zero, and this can be accomplished by changing the resultant angular velocity of the flywheel system by an amount $\Delta\bar{\omega}_F$ such that

$$J_F\Delta\bar{\omega}_F = J_1\bar{\omega}_1 \quad .$$

The change results in a reaction on the vehicle of an amount $-J_1\bar{\omega}_1$, so that the total angular momentum of the vehicle is

$$J_1\bar{\omega}_1 - J_1\bar{\omega}_1 = 0 \quad ,$$

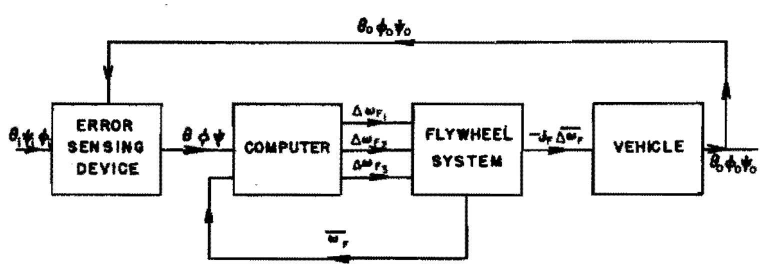
while the angular momentum of the flywheel system is

$$J_F\bar{\omega}_F + J_F\Delta\bar{\omega}_F = J_1\bar{\omega}_1 + J_F\bar{\omega}_F = \text{const} \quad .$$

Thus, by changing the angular momentum of the flywheel system, the vehicle angular velocity is reduced to zero in accordance with the principle of conservation of angular momentum.

Thus, by appropriate variations in velocity of the flywheel system, it should be possible to reduce the vehicle position and velocity errors to zero, with reference to the moving coordinate system.

A block diagram of the equipment necessary for such a control system is shown in Fig. 6. In this system, the error sensing device described previously, combines the desired heading and roll orientation θ_i, ψ_i, ϕ_i with the actual heading and roll position θ_o, ψ_o, ϕ_o, to give the vehicle attitude errors θ, ψ, ϕ. These errors are fed into a computer. At the same time the three component angular velocities of the flywheel system are also fed into the same computer. The computer determines the amount of moment necessary on each of the three flywheel shafts in order to produce the desired change in angular momentum of the flywheel system. The change of angular momentum is transferred to the vehicle as a reaction effect resulting in a change of attitude of the vehicle. When the attitude has reached the equilibrium condition, the flywheels are maintained at a constant angular velocity, and the vehicle remains at rest in the moving reference frame, or rotates with an angular velocity equal to the orbital angular velocity as referred to an inertial frame.



$\theta_i\ \psi_i\ \phi_i$ = DESIRED PITCH, YAW & ROLL ANGLES
$\theta_o\ \psi_o\ \phi_o$ = ACTUAL PITCH, YAW & ROLL ANGLES
$\theta\ \psi\ \phi$ = ERROR IN PITCH, YAW & ROLL ANGLES
$\overline{\omega}_f$ = VECTOR ANGULAR VELOCITY OF FLYWHEEL SYSTEM
$\Delta\omega_{f_1}, \Delta\omega_{f_2}, \Delta\omega_{f_3}$ = ANGULAR VELOCITY INCREMENTS OF FLYWHEELS
J_f = MOMENT OF INERTIA OF FLYWHEEL SYSTEM

ORBITAL CONTROL SYSTEM
FIG. 6

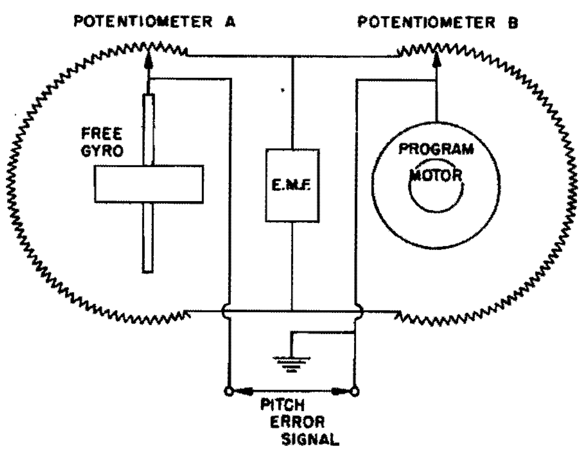
This control system requires more study, both from the theoretical and the practical point of view, and is described in this report schematically as a possible solution to the problem of attitude control in the orbit. Some calculations on the power requirements of the orbital control system are given in Appendix II.

EQUIPMENT

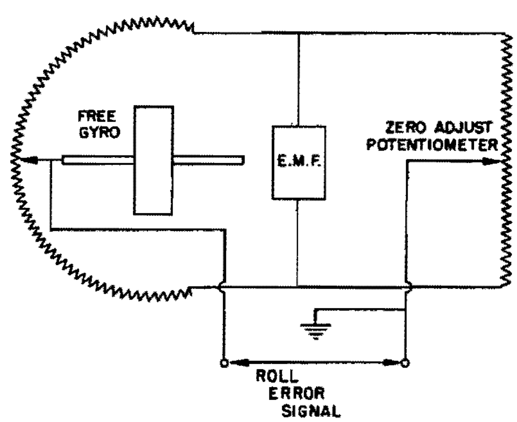
In the section on theory of the control system, very little has been said regarding the actual mechanisms which are required to accomplish the various functions indicated in the block diagrams of the control system. It is the purpose of the following section of the report to indicate, insofar as is possible at this time, the mechanical arrangement of the control system. As in the previous section, the problem is divided into two parts: first, the equipment necessary during the trajectory (powered flight and coasting); and second, the equipment necessary after the missile is in its orbit.

Equipment for Trajectory Control

The three error sensing devices for pitch, yaw, and roll are gyroscopic instruments with electrical pickoffs to indicate the error signals, and are similar in principle to those used in the V-2 control system. In the case of the pitch unit, a free gyro should be mounted in such a way that its axis of rotation remains parallel to the vertical at the launching position. The electrical circuit of the unit is shown in Fig. 7, in which a contact from the gyro rides on potentiometer A while a contact from the programming motor rides on potentiometer B. In the case of the V-2, only one potentiometer was used and both contacts were applied to it. With such a circuit, the voltage appearing across the output terminals is proportional to the difference in angular position of the two sliding contacts. Thus, if the programming motor causes its contact to move in accordance with the desired pitch orientation (referred to the original vertical), then the output signal is proportional to the angular error from the pitch program. The angular error signal applied to the control mechanism of Fig. 5 causes the vehicle (and potentiometer A) to rotate about the free gyro until its contact reaches a point symmetrical to that of potentiometer B. When this point is reached, the output of the device is zero and the vehicle is in the desired attitude. The two potentiometers used in the circuit should have at least 126° angular range to accommodate the pitch program specified, and the windings should be of very fine wire in order to give as continuous a variation of resistance as possible. This is particularly important if the derivative control is to be accomplished electrically.



PITCH ERROR SENSING DEVICE
FIG. 7



ROLL ERROR SENSING DEVICE
FIG. 8

The yaw and roll error sensing devices can both be operated from a single free gyro. The gyro is mounted with its axis normal to the desired plane of the trajectory, so that it is unaffected by the pitch program of the vehicle. Two contacts on this gyro move on potentiometers about the yaw and roll axes respectively. The electrical circuit for the roll axes is shown in Fig. 8; the circuit for the yaw axis being identical. In these circuits no programming device is necessary, and the only function of the potentiometer on the right is to give a convenient method of setting the null position of the system. This setting remains fixed during the trajectory and any deviation of the vehicle in roll or yaw results in a proportional signal, which causes the control system to operate and rotate the vehicle in such a way that the null positions of the roll and yaw potentiometers are kept under their respective contacts.

It may be necessary to use three rate gyros to indicate the rates of pitch, yaw, and roll in order to provide the necessary derivative control. However, further investigation is needed to determine whether it is more feasible to obtain such information from rate gyros or by differentiating the error signals electrically. If rate gyros are used, the block diagram of Fig. 5 would have to be altered by the addition of three feedback paths from the vehicle, which would carry the signals indicating $d\theta_o/dt$, $d\phi_o/dt$ and $d\psi_o/dt$. These signals would be combined with the programmed rate signals $d\theta_i/dt$, $d\phi_i/dt$ and $d\psi_i/dt$, to produce the error rate signals $d\theta/dt$, $d\phi/dt$, and $d\psi/dt$. The error rate signals would then be applied to the control amplifiers together with the original error signals and, with the exception of a difference in design of the three control amplifiers, the block diagram would be unaltered.

The pitch, roll, and yaw control amplifiers as shown in Fig. 5 are such that for an input voltage E_i, the output voltage E_o is given by the relation

$$E_o = A_o E_i + \frac{A_o K_i}{K_o}\frac{dE_i}{dt} + \frac{A_o K_{-1}}{K_o}\int E_i\,dt \quad , \tag{31}$$

or in operator form

$$E_o = \left(K_o' + K_1' p + \frac{K_{-1}}{p}\right) E_i \quad , \tag{32}$$

where A_o is the amplification factor of the amplifier and K_o, K_1, and K_{-1} are the system parameters determined in Eqs. (7).

One method by which this type of response might be obtained is by means of the two circuits shown in Figs. 9A and 9B. Circuit A produces a response of the type

$$E_o' = (C_o' + C_1' p)E_i' \quad , \tag{33}$$

while circuit B has a response

$$E_o'' = \left(C_o'' + \frac{C_1''}{p}\right)E_i'' \; . \tag{34}$$

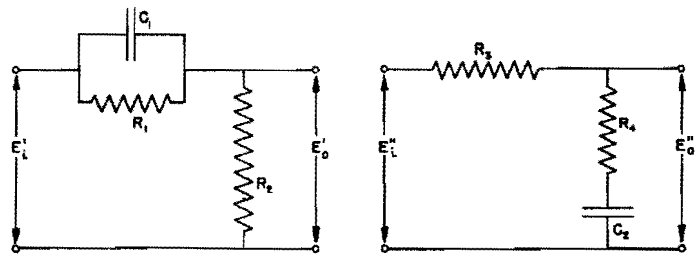


FIG. 9

If these two circuits are incorporated in the amplifier, in cascade, then the output is of the form

$$E_o = (C'_o + C'_1 p)\left(C''_o + \frac{C''_1}{p}\right)E_i = \left[(C'_o C''_o + C'_1 C''_1) + C''_o C'_1 p + \frac{C'_o C''_1}{p}\right]E_i \,, \qquad (35)$$

which is of the same form as the desired response of Eq. (32) if the following relations are satisfied

$$\begin{aligned} A_o &= K'_o = C'_o C''_o + C'_1 C''_1 \\ \frac{A_o K_1}{K_o} &= K'_1 = C''_o C'_1 \\ \frac{A_o K_{-1}}{K_o} &= K'_{-1} = C'_o C''_1 \quad . \end{aligned} \qquad (36)$$

However, this method involves certain approximations. In the first place, circuit A only approximates the response indicated in Eq. (33). Actually the response is given by

$$E'_o = \frac{C'_o + C'_1 p}{1 + A' p} E_i \qquad (37)$$

where A' represents the time delay of the system. It can be shown that as the amount of derivative control C'_1 is increased, the amount of time lag also increases. Thus, it is necessary to investigate whether, for the desired amount of derivative control, the resulting time lag becomes greater than can be tolerated in the overall system.

Similarly in circuit B the response of Eq. (34) is only approximate, the actual expression being

$$E_o'' = \frac{C_o'' + \dfrac{C_1''}{p}}{1 + \dfrac{C_2''}{p}} \quad . \tag{38}$$

By making C_2'' small it is possible to make (38) approximate (34) except at very low frequencies; if this degree of approximation is found to be unsatisfactory, it may be necessary to resort to some sort of feedback circuit in order to obtain a more exact integral control.

If, as indicated above, the time lag of the derivative circuit becomes excessive, it may be necessary to obtain a rate term directly from a rate gyro. This signal in combination with the error signal could produce the response of Eq. (33) directly and by applying the combination signal to circuit B in the manner described above, the desired response of Eq. (32) is obtained.

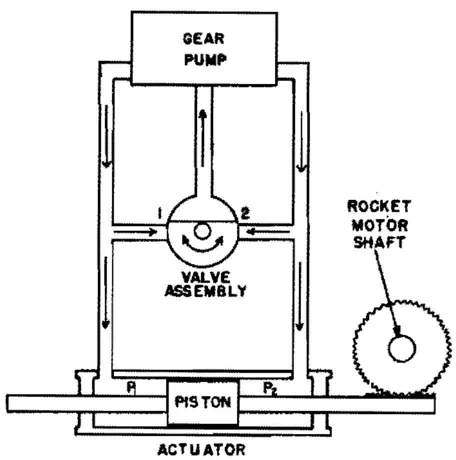
The deflection computer of Fig. 5 is a comparatively simple circuit which accepts the three output signals of the pitch, yaw, and roll control amplifiers S_P, S_Y, and S_R, and combines them in such a way as to produce the four outputs $S_P \pm S_R$ and $S_Y \pm S_R$.

The motor error sensing devices are simply a matter of connecting the input deflection signal δ_{in} and the actual motor deflection signal δ_{on} in such a way that the difference signal δ_n is produced, and electrically this would amount to measuring the difference of two voltages.

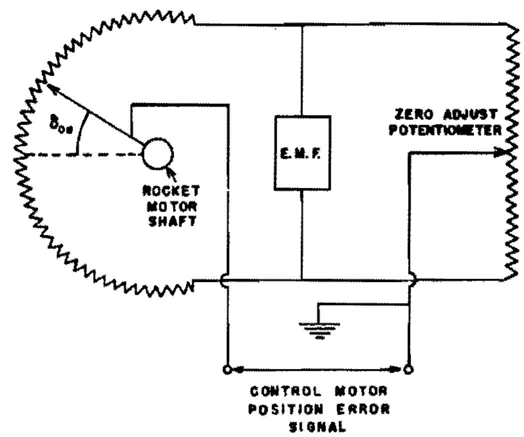
The design of the motor control amplifiers is parallel to that of the first set of control amplifiers and is subject to the same problems of supplying adequate derivative control. As in the previous case, if it cannot be done electrically, it will be necessary to detect the rate change of angular error in the motor position directly. The rate signal is combined with the error signal as in Eq. (33) and from here on the design is the same as before.

The servo motors to be used to drive the rocket control motors can be either electrical or hydraulic. However, since it is desirable to reduce the time lag in the control system to a minimum, it seems advisable to make use of a hydraulic servo motor of the type used in the V-2. In such a system, shown schematically in Fig. 10, a gear pump develops a pressure in the two output lines, and as long as valves #1 and #2 are both shut, equal pressures are exerted on the two ends of the piston in the lower cylinder and no motion occurs. If, however, valve #1 is opened, some of the liquid passes through and returns to the pump. As a result, the pressure P_1 decreases and the piston moves to the left. The motion is converted by a suitable mechanical system into a rotation of the rocket motor shaft.

To produce the feedback signal of the actual control motor deflection δ_{on} to the error sensing device, a potentiometer is mounted as shown in Fig. 11. A sliding contact fixed to the motor shaft moves along the potentiometer so that the output voltage is proportional to the angle, δ_{on}, between the instantaneous position of the shaft and its position when the contact is at the ground point of this potentiometer. The effective ground point can be adjusted to be the zero deflection position by means of the parallel potentiometer in the same manner as for the yaw and roll gyros.



HYDRAULIC SERVO MOTOR
FIG. 10



MOTOR POSITION ERROR SENSING DEVICE
FIG. 11

During the coasting period, the pitch and yaw error sensing is taken over by a molecular beam detector, see below, instead of the programmed gyro system. However, the roll control system remains on the roll gyro since the molecular beam detector gives no roll indication. The signal from the molecular beam detector is used to control the positions of the auxiliary control motors in the same manner as the gyro signal is used in the earlier part of the trajectory.

Thus, the control system during coasting is the same as during the powered flight, with the exception that the error signals are with respect to the flight path direction instead of the programmed position of the vehicle axis.

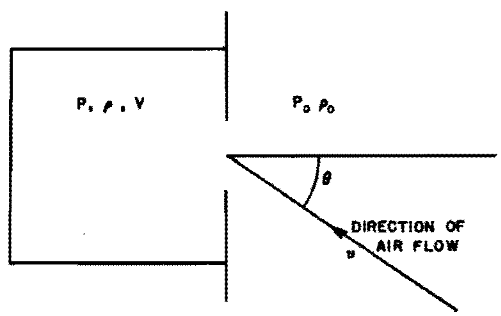
Orbital Control Equipment

The pitch and yaw error sensing device is based on the molecular beam principle, that if a stream of gas is incident on an orifice leading into a closed chamber, then the pressure developed in the chamber is proportional to the area of the orifice projected in a plane normal to the direction of motion of the molecules. Thus, in Fig. 12, if low density air is incident at an angle θ on the front of the chamber with an orifice of area A and a volume V, then it can be shown that the variation in chamber pressure is given by

$$\begin{aligned} P &= P_o + v\rho_o\left(\frac{2\pi R_o T}{M'}\right)^{1/2} \cos\theta \\ &\quad -\left(\frac{\pi}{2} < \theta < -\frac{\pi}{2}\right) \\ P &= P_o \\ &\quad \left(\frac{\pi}{2} < \theta < -\frac{\pi}{2}\right) \quad , \end{aligned} \tag{39}$$

and the time constant for the pressure buildup is given by

$$\tau_p = \frac{v}{A}\left(\frac{2\pi M'}{R_o T}\right)^{1/2} \quad . \tag{40}$$



P = CHAMBER PRESSURE
V = CHAMBER VOLUME
ρ = AIR DENSITY IN CHAMBER
P_o = EXTERNAL ATMOSPHERIC PRESSURE
ρ_o = EXTERNAL AIR DENSITY
v = ORBITAL VELOCITY OF SATELLITE
θ = ATTITUDE ANGLE OF SATELLITE

MOLECULAR BEAM ERROR SENSING DEVICE

FIG. 12

An evaluation of these two relations for the following normal orbit conditions

$$v = 7.62 \times 10^5 \text{ cm/sec}$$

$$\rho_o = 2.31 \times 10^{-14} \text{gms/cm}^3$$

$$R_o = 8.31 \times 10^7 \text{ ergs}/mol$$

$$T = 1940°K$$

$$M' = 24 \text{ gms}/mol$$

$$P_o = 1.17 \times 10^{-7} \text{ mm Hg}$$

gives

$$P = 1.17 \times 10^{-7} \, (1 + 23.2 \cos \theta) \text{ mm Hg} \tag{41}$$

$$\tau_p = 3.06 \times 10^{-5} \frac{V}{A} \frac{(\text{cm}^3)}{(\text{cm}^2)} \text{ sec} \cdot \tag{42}$$

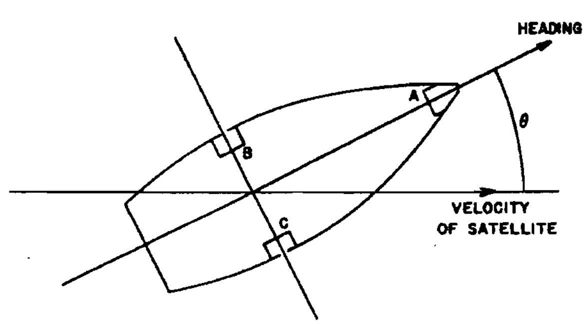
Thus, such an instrument gives a twenty fold variation in pressure from zero to ninety degrees, and the absolute pressures are in the range of those measurable by an ionization gauge. Also, the value of the time constant can be made as small as desirable by adjusting the chamber volume and aperture area.

In the vehicle five of these devices are used, one in the nose and four equally spaced around the circumference of the body as shown in Fig. 13. The analysis of the error measurement by this system is as follows. The pressures in the two detectors B and C are given by

$$\begin{aligned} P_C &= P_o + \Delta P \cos\left(\frac{\pi}{2} - \theta\right) & (0 < \theta < \pi) \\ &= P_o & (-\pi < \theta < 0) \\ P_B &= P_o & (0 < \theta < \pi) \\ &= P_o - \Delta P \cos\left(\frac{\pi}{2} + \theta\right) & (-\pi < \theta < 0) \end{aligned} \tag{43}$$

where

$$\Delta P = v \rho_o \left(\frac{2\pi R_o T}{M'}\right)^{1/2} \cdot \tag{44}$$



LOCATION OF MOLECULAR BEAM DETECTORS
FIG. 13

If the signals from the two detectors are proportional to the chamber pressures, then the two signals are given by

$$\begin{aligned} E_C &= kP_C = kP_o + k\Delta P \cos\left(\frac{\pi}{2} - \theta\right) && (0 < \theta < \pi) \\ &= kP_o && (-\pi < \theta < 0) \\ E_B &= kP_B = kP_o - k\Delta P \cos\left(\frac{\pi}{2} + \theta\right) && (-\pi < \theta < 0) \\ &= kP_o && (0 < \theta < \pi) \end{aligned} \quad (45)$$

If these two signals are combined to give their difference, the net signal becomes

$$\begin{aligned} E_C - E_B &= + k\Delta P \cos\left(\frac{\pi}{2} - \theta\right) && (0 < \theta < \pi) \\ &= + k\Delta P \cos\left(\frac{\pi}{2} + \theta\right) && (-\pi < \theta < 0) \end{aligned} \quad , \quad (46)$$

which reduces to a single expression

$$E_C - E_B = k\Delta P \sin\theta \quad . \quad (47)$$

This expression gives unambiguous attitude indications for values of θ between $\pm\,\pi/2$. Thus, it is necessary to include some mechanism which makes this detector inoperative when the pressure in detector A reduces to P_o. In this way the detector gives a fairly linear error indication at small error angles and can handle errors up to ninety degrees, but beyond this point it is blind. It seems reasonable to expect that even under extreme conditions, the control system should be able to keep the vehicle heading within this operating range of the detector.

A similar set of detectors is used in the control of yaw with detector A used commonly by both the pitch and yaw systems.

The roll error detection system consists of a flux gate compass which determines the deviation between the position of the yaw axis and the component of the earth's magnetic field perpendicular to the orbit. This error signal is used at specific points along the trajectory to reset the zero of the roll gyro which determines the roll attitude error during the orbit. Even if this zero adjustment is made only once per orbital period, the drift of the roll gyro between settings is of the order of 5° which is within the allowable attitude error in the orbit.

The actual mechanism of the computer requires further analysis of the rotational behavior of the combined vehicle and flywheel system before any definite design can be suggested.

The flywheel system of the vehicle consists of a set of three flywheels with mutually perpendicular axes parallel to the pitch, yaw, and roll axes of the vehicle. A discussion of these flywheels is included in Appendix II of this report.

CONCLUSIONS

The proposed control system for the satellite rocket operates in the following manner. During the period from the start of the trajectory through the first and second burning periods and the portion of the third burning period prior to coasting, the vehicle follows a predetermined heading program. Deviations from the program are determined by means of a pitch gyro and a yaw-roll gyro and appropriate correcting moments are applied to the vehicle by deflections of auxiliary rocket control motors. During the coasting period, no appreciable alteration in the flight path is possible, and the problem becomes that of keeping the angle of attack zero with respect to the existing flight path. For this reason, the pitch and yaw gyros are replaced by a molecular beam detector which measures the deviation of the vehicle attitude from a zero angle of attack. The roll detection system is still based on the roll gyro used in the earlier part of the trajectory, and the correction moments are supplied by the auxiliary rocket motors as before. This second control combination continues in operation during the coasting and the short burning period after coasting. Thus, the vehicle enters its orbit with a zero angle of attack as the rocket power is shut off. After the vehicle is in its orbit, the molecular beam detector for pitch and yaw continues to operate. Likewise, the roll gyro is also used, but its zero position is adjusted at specific points on the orbit with reference to the earth's field so that its cumulative drift error does not exceed the tolerance of $\pm 5°$. The attitude correction of the vehicle during the orbit is accomplished by producing a rotational reaction effect on the vehicle by rotating a flywheel system mounted in it, the amount of flywheel rotation being determined by the signals from the pitch, yaw, and roll detecting devices.

The trajectory control system described above appears to be adequate to satisfy the requirements set down in the Introduction. While the actual numerical evaluation of the control system parameters must await the final trajectory and structural data, the method of evaluating these parameters gives a system which is stable throughout the trajectory despite the variation of such quantities as moment of inertia of the vehicle, position of center of gravity, aerodynamic restoring moment, and aerodynamic damping moment. Also, the magnitude of the control moments available in each stage is more than adequate to overcome the aerodynamic moments resulting from the tilt program, and is adequate to give a response time of the order of 4 seconds in the correction of an error in the vehicle heading. In addition to satisfying these basic requirements, the system has the advantage that under the action of a constant external moment the integral action of the control causes the error to reduce to zero; also, during the transient motion due to a correction in heading, the net effect of lateral thrust of the various rocket motors is to produce no lateral velocity and a negligible lateral displacement. This effect is particularly important in a programmed trajectory of the type proposed for this vehicle.

Certain parts of the trajectory control system require further study before a final design is determined. In particular, the time lag between error signal and application of control moment should be studied in relation to its effect on the stability of the vehicle. This study would give some idea of the permissible time lag in the overall system and the amount of compensation necessary. Also, a study of the available servo components in regard both to weight, performance, and accuracy might result in some modification in the finally selected system parameters.

The orbital control system as described in this report is in somewhat qualitative form since it involves more unknowns than the trajectory control system. Thus, further study and development are needed on the problem with regard to the molecular beam detection device for pitch and yaw, the magnetic roll detection device, and the computer which converts these error signals into velocity increments for the flywheel system.

References

[1]Flight Mechanics of a Satellite Rocket, RA-15021, Project RAND, Douglas Aircraft Company, Inc., February 1, 1947

[2]Routh, E.J., *Advanced Rigid Dynamics*. London, 1905

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APPENDIX I.

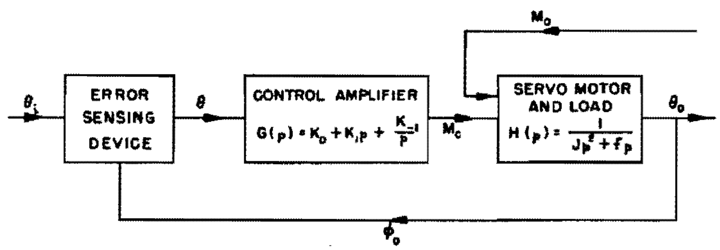
SERVOMECHANISM THEORY

The servomechanism which has been considered for the control system of the satellite rocket is one which responds to error, derivative, and integral control as indicated in the body of the report. This appendix includes the theory of this mechanism upon which its selection was based.

TRANSIENT RESPONSE OF SERVOMECHANISMS

The block diagram of the servomechanism is shown in Fig. 14. In the system, the error sensing device takes the input angle θ_i and the output angle θ_o and combines them to produce the error angle θ in accordance with the relation

$$\theta = \theta_i - \theta_o . \tag{48}$$



θ_i = INPUT SIGNAL
θ_o = OUTPUT SIGNAL
θ = ERROR SIGNAL
M_c = CONTROL MOMENT
M_o = EXTERNAL MOMENT
$G(p)$ = CONTROL OPERATOR
$H(p)$ = CONTROLLED SYSTEM OPERATOR

BLOCK DIAGRAM OF A SERVOMECHANISM

FIG. 14

The error signal is converted by the control amplifier into a control moment by the relation

$$M_c = K_o\theta + K_1 \frac{d\theta}{dt} + K_{-1} \int_o^t \theta dt . \tag{49}$$

The moment is applied to the servo motor and load. However, in addition to the control moment, there may be some external moment M_o applied to the mechanical system. Thus the equation of motion of the output is given by

$$J \frac{d^2\theta_o}{dt^2} + f \frac{d\theta_o}{dt} = M_c + M_o \, . \tag{50}$$

To study the transient response of the system, it is more convenient to use an expression relating the error angle to the input angle. This can be found by combining Eqs. (48), (49), and (50) so that

$$J \frac{d^2\theta_i}{dt^2} + f \frac{d\theta_i}{dt} = J \frac{d^2\theta}{dt^2} + (f + K_1) \frac{d\theta}{dt} + K_o\theta + K_{-1} \int_0^t \theta dt + M_o \, . \tag{51}$$

For convenience the following parameters are introduced:

$$\begin{aligned} \omega_n &= \sqrt{\frac{K_o}{J}} \\ \gamma &= \frac{f}{2\sqrt{K_o J}} \\ \zeta &= \frac{f + K_1}{2\sqrt{K_o J}} \\ S &= \frac{K_{-1}}{K_o}\sqrt{\frac{J}{K_o}} \\ \alpha_o &= \frac{M_o}{K_o} \, . \end{aligned} \tag{52}$$

Substitution of these quantities gives

$$\frac{d^2\theta_i}{dt^2} + 2\gamma\omega_n \frac{d\theta_i}{dt} = \frac{d^2\theta}{dt^2} + 2\zeta\omega_n \frac{d\theta}{dt} + \omega_n^2\theta + S\omega_n^3 \int \theta dt + \alpha_o\omega_n^2 \, . \tag{53}$$

To solve the equation, the Laplace transform is made and the resulting transform equation is solved for $\overline{\theta}$ as follows:

$$\overline{\theta} = \frac{p(p^2 + 2\gamma\omega_n p)\overline{\theta}_i - (p^2 + 2\gamma\omega_n p)\theta_i(0) - \dot{\theta}_i(0) + \dot{\theta}(0) + S\omega_n^3\theta^{-1}(0)}{p^3 + 2\zeta\omega_n p^2 + \omega_n^2 p + S\omega_n^3} + \tag{54}$$

$$+ \frac{(p^2 + 2\zeta\omega_n p)\theta(0) - p\omega_n^2\overline{\alpha}_o}{p^3 + 2\zeta\omega_n p^2 + \omega_n^2 p + S\omega_n^3} . \qquad (54 \text{ cont'd})$$

There are two special cases of this equation which are of particular interest. The first is the response to a step function input; while the second is the response to a suddenly applied external moment.

The response to a step function can be obtained by the following substitution:

Initial conditions

$t = 0$

$$\theta_i(0) = \phi$$

$$\dot{\theta}_i(0) = 0$$

$$\theta(0) = \phi$$

$$\dot{\theta}(0) = 0$$

$$\theta^{-1}(0) = 0$$

Input function

$$\theta_i(t) = 0 \qquad t < 0$$

$$= \phi \qquad t > 0$$

$$\overline{\theta}_i = \frac{\phi}{p}$$

External moment

$$\alpha_o(t) = 0$$

$$\overline{\alpha}_o = 0$$

Substitution of these conditions in Eq. (54) gives the following:

$$\overline{\theta} = \frac{p^2 + 2\zeta\omega_n p}{p^3 + 2\zeta\omega_n p^2 + \omega_n^2 p + S\omega_n^3} \times \phi . \qquad (55)$$

In the case of a suddenly applied external moment M_o with no change of input the following conditions must be applied:

Initial conditions

$$t = 0$$

$$\theta_i(0) = 0$$

$$\dot{\theta}_i(0) = 0$$

$$\theta\ (0) = 0$$

$$\dot{\theta}\ (0) = 0$$

$$\theta^{-1}(0) = 0\ .$$

Input function

$$\theta_i(t) = 0$$

$$\overline{\theta}_i = 0\ .$$

External moment

$$\alpha_o(t) = 0 \qquad t < 0$$

$$\alpha_o = \alpha_o \qquad t > 0$$

$$\overline{\alpha}_o = \frac{\alpha_o}{p}\ .$$

Substitution of these quantities gives the relation

$$\overline{\theta} = -\frac{\alpha_o \omega_n^2}{p^3 + 2\zeta\omega_n p^2 + \omega_n^2 p + S\omega_n^3}\ . \qquad (56)$$

Eqs. (55) and (56) give the Laplace transform of the error angle for the two cases considered. In order to obtain a solution of θ as a function of time, it is necessary to obtain the roots of the cubic in the denominator and in the general case this is a rather cumbersome procedure. However, it is of interest to consider the case in which all of the roots are equal. The condition occurs when

$$\zeta = \frac{\sqrt{3}}{2}$$

$$S = \frac{\sqrt{3}}{9}$$

and all of the roots of the cubic are then equal to $\omega_n/\sqrt{3}$. Under these conditions, the inverse Laplace transformation gives the following solution for the error angle after a step input.

$$\theta = \phi e^{-\frac{\omega_n}{\sqrt{3}}t}\left(1 + \frac{\sqrt{3}}{3}\omega_n t - \frac{1}{3}\omega_n^2 t^2\right). \tag{57}$$

Similarly, the solution in the case of a suddenly applied external moment becomes

$$\theta = -\frac{M_o}{2J}t^2 e^{-\frac{\omega_n}{\sqrt{3}}t} . \tag{58}$$

In general, the time constant for an integral control system can be expressed as

$$\tau = \frac{2}{S\omega_n} , \tag{59}$$

and for the particular system discussed, ω_n is given by

$$\omega_n = \frac{6\sqrt{3}}{\tau} . \tag{60}$$

Thus, the two error expressions become

$$\theta = \phi e^{-6\frac{t}{\tau}}\left[1 + 6\frac{t}{\tau} - 36\left(\frac{t}{\tau}\right)^2\right] \tag{57a}$$

$$\theta = -\frac{M_o\tau^2}{2J}\left(\frac{t}{\tau}\right)^2 e^{-6\frac{t}{\tau}} . \tag{58a}$$

In future work it will be useful to have an expression for the control moment required from the servo motor in performing its control function. This can be obtained by substituting Eqs. (59a) and (58a) in Eq. (49). In making the substitution, it is assumed that the viscous friction is zero so that

$$\zeta = \frac{K_1 + f}{2\sqrt{JK_o}} = \frac{K_1}{2\sqrt{JK_o}} = \frac{\sqrt{3}}{2} .$$

With this assumption, the control moment expressions become:

Step Function Input

$$M_c = \frac{108\, J\phi}{\tau^2} e^{-6\frac{t}{\tau}}\left[12\left(\frac{t}{\tau}\right)^2 - 10\frac{t}{\tau} + 1\right] \tag{61}$$

Suddenly Applied External Moment

$$M_c = M_o\left\{\left[18\left(\frac{t}{\tau}\right)^2 - 12\frac{t}{\tau} + 1\right] e^{-6\frac{t}{\tau}} - 1\right\} \tag{62}$$

Plots of the two transient responses of Eqs. (57a) and (58a) are shown in Figs. 15 and 16, while the corresponding moment relations, Eqs. (61) and (62) are shown in Figs. 17 and 18. From Eqs. (61) and (62) it can be shown that the maximum moments required from the motor are given by

$$M_m = \frac{108\, J\phi}{\tau^2} \tag{61a}$$

$$M_m = 1.206\, M_o \quad . \tag{62a}$$

Also, from Eq. (58a) it can be shown that the maximum error angle, due to a suddenly applied external moment, is given by

$$\theta_m = -\,.0075\,\frac{M_o\,\tau^2}{J}\ \text{(radians)}. \tag{63}$$

In the case of a step function input, it can be shown from Eq. (57a) that the maximum overshoot occurs at one-half the response time and has a magnitude given by

$$\theta = -\,.249\,\phi\,. \tag{64}$$

The above discussion gives the characteristic responses of the servo system in terms of the time constant τ, the moment of inertia J, the initial input amplitude ϕ, and the external moment M_o. It is also necessary to specify the constants K_o, K_1, and K_{-1} in terms of some or all of these same quantities. This can be done by means of the relations (52) and (59) using the selected values for S and ζ

$$K_o = \frac{4\,J}{S^2\,\tau^2} = 108\,\frac{J}{\tau^2}$$

$$K_1 = \frac{4\,\zeta\,J}{S\tau} - f = 18\,\frac{J}{\tau} - f \tag{65}$$

$$K_{-1} = \frac{8\,J}{S^2\,\tau^3} = \frac{216\,J}{\tau^3} \quad .$$

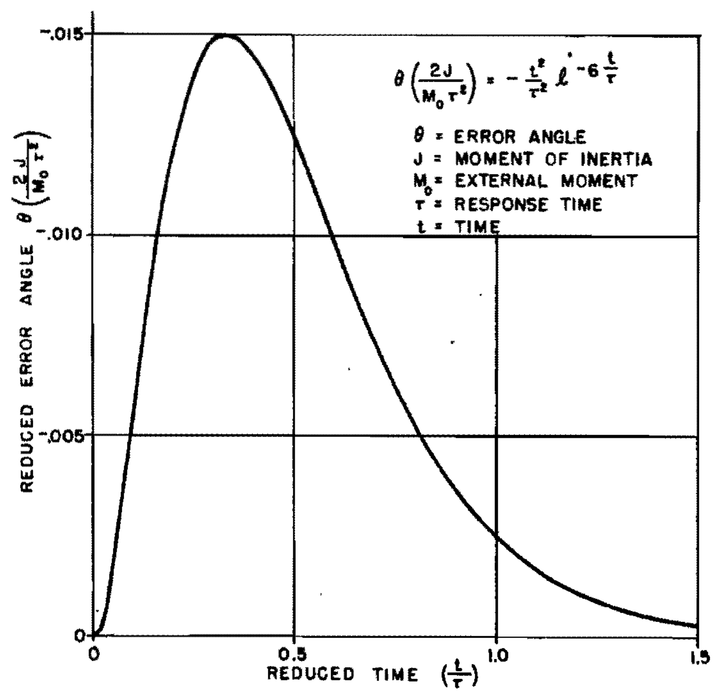
STABILITY OF SERVOMECHANISMS

In the preceding discussion of the transient response of the servo system, no consideration is given to the problem of whether the response is stable. While the particular numerical case which is considered does give a stable response, it is necessary to know what variation in the system parameters can be tolerated without introducing instability. This information can be obtained either from Routh's rule for stability[2] or from Nyquist's criterion[3]. Both of these methods are discussed below.

In Eqs. (55) and (56) giving the Laplace transform of the error angle, the cubic expression

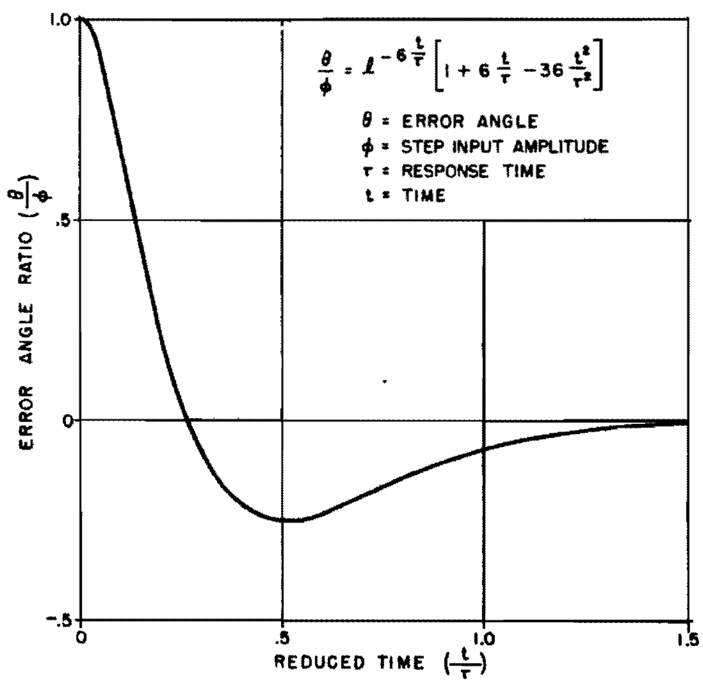
$$f(p) = p^3 + 2\,\zeta\,\omega_n\,p^2 + \omega_n^2\,p + S\,\omega_n^3 \tag{66}$$

For references see page 46



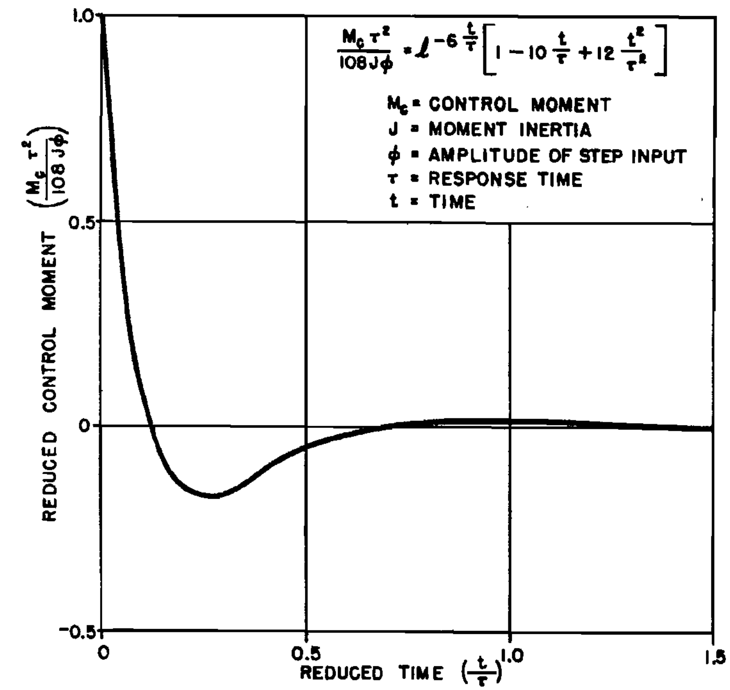
RESPONSE OF INTEGRAL SERVOMECHANISM
TO SUDDEN EXTERNAL MOMENT

FIG. 16



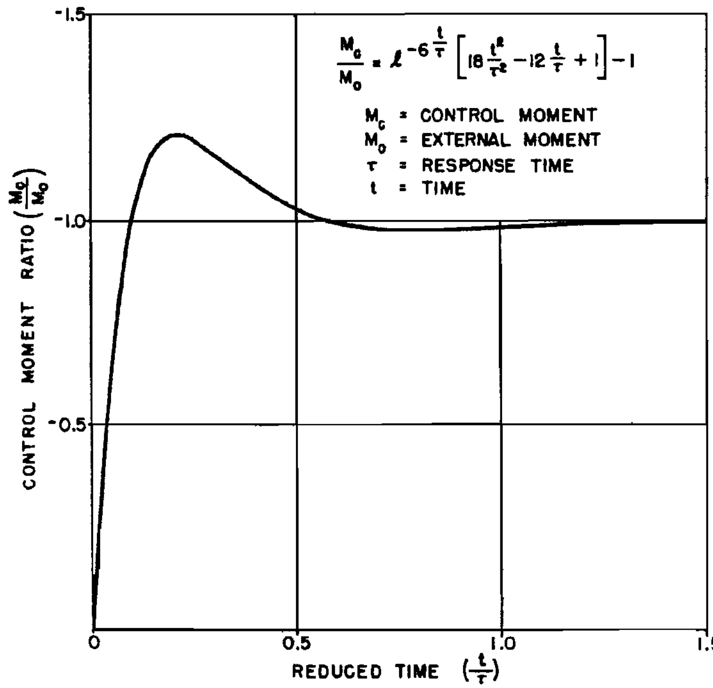
TRANSIENT RESPONSE
OF INTEGRAL SERVOMECHANISM

FIG. 15



CONTROL MOMENT DURING TRANSIENT RESPONSE OF INTEGRAL SERVOMECHANISM

FIG. 17



CONTROL MOMENT DURING CORRECTION FOR SUDDEN EXTERNAL MOMENT

FIG. 18

appears in the denominator. In order that the system be stable, it is necessary that this polynomial have all negative real roots or roots with negative real parts. Then, when the inverse Laplace transformation is made, these roots appear in the exponents of e and give factors of the form e^{-at} which reduce the amplitude of the transient to zero at time $t = \infty$.

Routh shows that if a cubic is of the form

$$Ap^3 + Bp^2 + Cp + D = 0 ,$$

then, the condition for all roots to have negative real parts is that A, B, C, and D are all positive and that

$$BC > AD . \tag{67}$$

If the condition is applied to Eq. (66) it gives the result that for stability

$$\begin{aligned} \zeta &> 0 \\ \omega_n &> 0 \\ S &> 0 \end{aligned} \tag{68}$$

and

$$2\zeta > S . \tag{69}$$

In the particular system considered in the previous section, condition (68) is satisfied since

$$\zeta = \frac{\sqrt{3}}{2} > 0$$

$$S = \frac{\sqrt{3}}{9} > 0$$

$$\omega_n > 0$$

and condition (69) is also satisfied since

$$2\zeta = \sqrt{3} > \frac{\sqrt{3}}{9} = S .$$

The above is in agreement with the resulting transient responses obtained in the previous section.

While Routh's rule for stability of a cubic is comparatively simple, the corresponding relations for higher degree functions become increasingly complex and do not adapt themselves to design problems. Thus, it is more convenient to make use of the Nyquist criterion commonly used in feedback amplifier design. To apply the criterion, it is necessary to obtain the complex expression for the response of the system to a sinusoidal input. This can be done as follows:

In operator form the equations of the system are

$$\begin{aligned} \theta &= \theta_i - \theta_o \\ M_c &= G(p)\theta \\ \theta_o &= H(p)M_c \end{aligned} \tag{70}$$

By eliminating θ and M_c from these equations the following relation between θ_i and θ_o is obtained

$$\theta_o = \frac{G(p)H(p)}{1 + G(p)H(p)}\theta_i\ , \tag{71}$$

where p is the operator $\frac{d}{dt}$.

For a sinusoidal input p is replaced by $j\omega$ so that

$$\frac{\theta_o}{\theta_i}(j\omega) = \frac{G(j\omega)H(j\omega)}{1 + G(j\omega)H(j\omega)}\ . \tag{72}$$

The relation is the complex frequency response of the system, and if for a frequency ω_1

$$G(j\omega_1)H(j\omega_1) = -1\ , \tag{73}$$

then, at this frequency the amplitude characteristic has an infinite value, which corresponds to a sustained oscillation at the resonant frequency of the system, and represents a limiting condition between a stable and an unstable response.

Nyquist's criterion makes use of the above property of the transfer function $G(j\omega)H(j\omega)$ in the following way. The transfer function and its complex conjugate (mirror image through the real axis) is plotted in the complex plane. If the resulting curve does not completely enclose the point -1 on the real axis, the system is stable, while if the curve does enclose the point, the system is unstable. In accordance with Eq. (73), if the curve passes through the point -1, the system undergoes a sustained oscillation with zero input.

If the above analysis is applied to the servo system considered above, the following results are obtained

$$\begin{aligned} G(p) &= K_o + K_1 p + \frac{K_{-1}}{p} \\ H(p) &= \frac{1}{Jp^2 + fp}\ . \end{aligned} \tag{74}$$

Thus, the transfer function becomes

$$G(p)H(p) = \frac{K_o + K_1 p + \dfrac{K_{-1}}{p}}{Jp^2 + fp}$$

$$G(p)H(p) = \frac{\omega_n^2 p + 2\delta\omega_n p^2 + S\omega_n^3}{p(p^2 + 2\gamma\omega_n p)}\,, \qquad (75)$$

where

$$\delta = \zeta - \gamma \ .$$

By letting $p = j\omega$ and rationalizing the resulting equation the transfer function becomes

$$G(j\omega)H(j\omega) = -\frac{\left(\frac{\omega}{\omega_n}\right)^2 + 2\gamma\left[S - 2\delta\left(\frac{\omega}{\omega_n}\right)^2\right]}{\left(\frac{\omega}{\omega_n}\right)^2\left[\left(\frac{\omega}{\omega_n}\right)^2 + 4\gamma^2\right]} - j\,\frac{2\gamma - \left[S - 2\delta\left(\frac{\omega}{\omega_n}\right)^2\right]}{\frac{\omega}{\omega_n}\left[\left(\frac{\omega}{\omega_n}\right)^2 + 4\gamma^2\right]} \qquad (76)$$

The stability conditions can be obtained from this relation by applying Nyquist's criterion which specifies that the transfer function must cross the real axis to the left of the point −1. The frequency at which the crossing occurs is given by setting the imaginary part equal to zero.

$$2\gamma - \left[S - 2\delta\left(\frac{\omega}{\omega_n}\right)^2\right] = 0\,,$$

thus

$$\left(\frac{\omega}{\omega_n}\right)^2 = \frac{S - 2\gamma}{2\delta} \ .$$

Substitution of this frequency in the real part, and applying the condition that the real part be less than −1, gives

$$-\frac{\left(\frac{\omega}{\omega_n}\right)^2 + 2\gamma\left[S - 2\delta\left(\frac{\omega}{\omega_n}\right)^2\right]}{\left(\frac{\omega}{\omega_n}\right)^2\left[\left(\frac{\omega}{\omega_n}\right)^2 + 4\gamma^2\right]} \leqq -1$$

or

$$\frac{2\delta}{S - 2\gamma} \geqq 1$$

$$2(\delta + \gamma) \geqq S$$

$$2\zeta \geqq S$$

which corresponds with that obtained from Routh's rule for stability.

In the section on the transient response of the servo system, it was only necessary to specify the values of $\zeta = \sqrt{3}/2$ and $S = \sqrt{3}/9$. However, in order to plot the transfer function, it is also necessary to indicate how much of ζ is due to frictional damping (γ) and how much is due to derivative control (δ).

Two cases are considered, the first in which all of ζ is due to frictional damping, and the second in which all of ζ is due to derivative control.

In the first case

$$S = \frac{\sqrt{3}}{9}$$

$$\zeta = \frac{\sqrt{3}}{2}$$

$$\delta = 0$$

$$\gamma = \frac{\sqrt{3}}{2}$$

and the transfer function is

$$G(j\omega)H(j\omega) = -\frac{3\left(\frac{\omega}{\omega_n}\right)^2 - 1}{3\left(\frac{\omega}{\omega_n}\right)^2\left[\left(\frac{\omega}{\omega_n}\right)^2 + 3\right]} - j\,\frac{8}{9}\sqrt{3}\,\frac{1}{\frac{\omega}{\omega_n}\left[\left(\frac{\omega}{\omega_n}\right)^2 + 3\right]}\,. \tag{77}$$

In the second case

$$S = \frac{\sqrt{3}}{9}$$

$$\zeta = \frac{\sqrt{3}}{2}$$

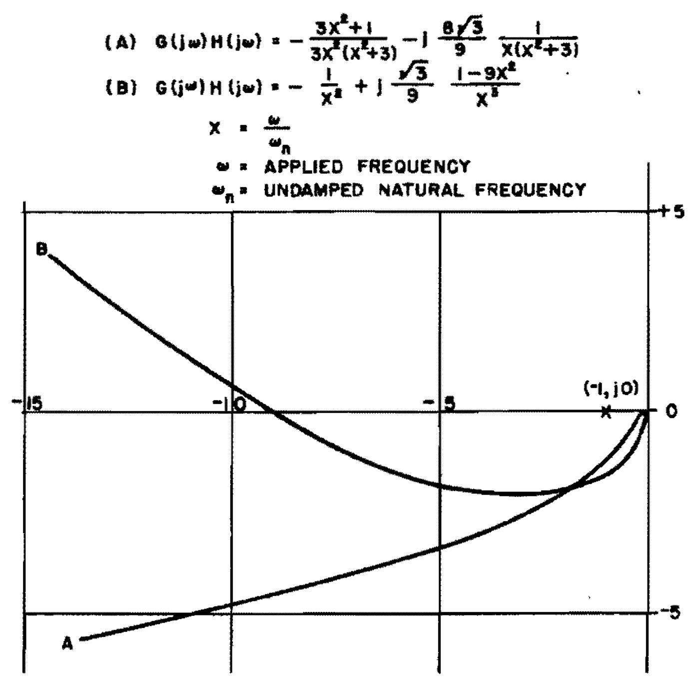
$$\delta = \frac{\sqrt{3}}{2}$$

$$\gamma = 0$$

and the transfer function is

$$G(j\omega)H(j\omega) = -\frac{1}{\left(\frac{\omega}{\omega_n}\right)^2} + j\frac{\sqrt{3}}{9}\,\frac{1 - 9\left(\frac{\omega}{\omega_n}\right)^2}{\left(\frac{\omega}{\omega_n}\right)^3} \quad . \tag{78}$$

The Nyquist diagrams for Eqs. (77) and (78) are plotted in Fig. 19 and it is seen that both cases are stable. Thus, it is possible to design a stable system such that the sum of the frictional damping and the derivative control gives a ζ of $\sqrt{3}/2$.



A. FRICTIONAL DAMPING
B. DERIVATIVE CONTROL DAMPING

NYQUIST DIAGRAMS FOR INTEGRAL SERVOMECHANISM

FIG. 19

SUMMARY

In order to design a servomechanism of the type discussed in the foregoing analysis, it is necessary to specify the following quantities:

1. Moment of Inertia (J) of the moving system
2. Viscous Friction (f) of the moving system
3. Desired Time Constant (τ) of the system
4. Maximum Error Angle (ϕ) to be controlled
5. Maximum External Moment (M_0) to be controlled

From these quantities it is possible by use of Eqs. (61a), (62a), and (65) to specify the following system parameters

1. Maximum Control Moment (M_m)
2. Error Control Constant (K_0)
3. Derivative Control Constant (K_1)
4. Integral Control Constant (K_{-1})

If the system to be controlled has a moment of inertial and viscous friction which varies with time, it is not possible to fix the values of ζ and S at $\sqrt{3}/2$ and $\sqrt{3}/9$. Instead, these values are taken as average throughout the time of operation, taking care that the variations in S and ζ do not violate the stability condition at any time during the operation.

APPENDIX II

ATTITUDE CONTROL DURING COASTING AND WHILE IN ORBIT*

Attitude control of the satellite rocket during the coasting period and during the orbit will be discussed separately as the two regimes represent different problems, due mainly to the duration of operation involved.

A. COASTING ATTITUDE CONTROL

Coasting attitude control is the method of maintaining the satellite rocket in a given orientation relative to its flight path, during the coasting period inserted in the final stage, in an atmosphere of such low density that aerodynamic forces are negligible.

1. The satellite rocket cannot be permitted to rotate about any axis during this period for four reasons:

   a. The thrust of the main rocket motor, which is turned on for a short period after coasting, must be along the flight path to achieve the desired result of putting the satellite on its orbit[1].

   b. The auxiliary power plant, if operated as a closed mercury vapor turbine system or as a thermo-pile, requires a 'cold' side of the vehicle. In the first case this requirement is for the condenser, in the second, for the cold junction of the thermo-pile. This cold side can only be obtained by radiation from the satellite[2].

   c. For minimum power requirements for the telemetering system, satellite mounted directional antennas are used[3].

   d. It will be of advantage to know the orientation of the satellite for the reduction of scientific data gathered by instruments mounted in the satellite.

2. Control while coasting is maintained by allowing the control rocket motors (which represent approximately 1/60 of the total thrust of the third stage) to operate continuously during the coasting period. The total amount of fuel required is not affected by burning in this manner, but rather the length of burning of the main motor before and after coasting is adjusted so that the same performance results. This is discussed in the chapter on aerodynamics[1].

B. ORBIT ATTITUDE CONTROL

In the orbit the problem is the same as during coasting, except that the control rocket motors are no longer operating.

*Written by J.O. Crum.

1. The satellite rocket cannot be permitted to roll about any axis while in the orbit for the same reasons as listed in A.1 with the exception of a.

2. Control Systems Considered For the Orbit

As under these conditions the satellite rocket is an isolated body, its attitude can only be changed by a transfer of momentum between components of the vehicle itself. To accomplish this, there are several possibilities available:

a. Continuous or intermittent operation of the control rocket motors of the satellite rocket while in the orbit. This requires large amounts of fuel for continuous, fine control. The result is a reduction in payload.

b. An arrangement of small solid propellant rockets of graduated sizes about the surface of the satellite rocket to be fired singularly or in any combination required to accomplish the desired change in attitude. This was ruled out on the basis of complexity of control, limited fineness of control, and limited time during which control may be had (i.e., weight).

c. Air or gas jets suitably located about the satellite rocket. Here again, weight limits the duration of continuous control.

d. A system of three flywheels mutually at right angles in the satellite rocket, each flywheel being able to rotate about only one axis relative to the satellite. The flywheels are to be driven by electric motors. The power for the motors is to come from the satellite auxiliary power plant. This device appears most favorable from a weight viewpoint, as well as continuous, fine control. This is the method chosen.

e. A sphere capable of rotation about any axis and driven by a rotating magnetic field, the direction of rotation of which is determined by the correction required. Power is again to come from the auxiliary power plant. This method was set aside as a longer term development problem because of the unknown action on the sphere of the magnetic field as it changed its direction of rotation.

3. Method of Control in Orbit

Control will be achieved by transferring momentum to or from three flywheels initially at rest relative to the satellite rocket. These flywheels are mounted in the satellite mutually at right angles, each being capable of rotation about one axis only relative to the satellite.

This system requires a computer capable of solving continuously the following problem. Given: a required change of momentum of the satellite in a given direction. In addition the instantaneous angular velocity of each of the flywheels is given. Find: the required change of momentum of each of the flywheels taking account of the gyroscopic effect.

4. Power Required for Operation of Flywheel

As a first approximation to the power required to drive the flywheels, consider the following:

Given: a single flywheel, free to rotate about the pitch axis, initially at rest in the satellite. The satellite has arrived on the orbit with one side parallel to the surface of the earth.

Find: horsepower necessary to drive this flywheel to keep the given side of the satellite pointed toward the earth, this momentum change to occur in τ seconds.

Capital letters refer to vehicle
Small letters refer to flywheel
$_1$ refers to initial conditions
$_2$ refers to final conditions
I, i = moment of inertia
Ω, ω = angular velocity
P = power
t, τ = time

Angular momentum

$$\text{Initial} = I\,\Omega\,,$$
$$\text{Intermediate} = I\,\Omega + i\,\omega = I\,\Omega\,,$$
$$\text{Final} = i\,\omega_2 = I\,.$$

Energy

$$\text{Initial} = \frac{1}{2} I\,\Omega_1^{\,2}$$
$$\text{Intermediate} = \frac{1}{2} I\,\Omega^2 + \frac{1}{2}\dot{\omega}^2 i = \frac{1}{2} I\,\Omega_1^{\,2} + Pt$$
$$\text{Final} = \frac{1}{2}\,i\,\omega_2^{\,2} = \frac{1}{2} I\,\Omega_1^{\,2} + P\,\tau\,.$$

Then

$$\omega_2 = \frac{I}{i}\Omega_1$$

and

$$\text{hp} = \text{horsepower} = \frac{P}{550} = \frac{1}{2} \times \frac{1}{550} \times \frac{\Omega_1^{\,2}}{\tau}\left[i\left(\frac{I}{i}\right)^2 - I\right]\,.$$

Example:

Minimum power requirement will be to keep one side of the satellite rocket facing the earth (i.e., impart to the satellite an angular velocity of one revolution per 1-1/2 hours, which is the time required for the satellite to make one revolution about the earth). This momentum change is to be accomplished in 10 minutes. *(I/i = 1750 and i = 0.2 slug ft^2, are typical values.)*

$$= \frac{1}{2} \times \frac{1}{550} \times \left(\frac{2\pi}{1\frac{1}{2} \times 60 \times 60}\right)^2 \times \frac{1}{600}\left[0.2(1750)^2 - 350\right]$$

$$= 2.15 \times 10^{-12}\ (612{,}000)$$

$$= 1.3 \times 10^{-6} \text{ horsepower}$$

Assuming an efficiency of 1%, due to the small energy to be transferred, the horsepower required is still only of the order of 10^{-4}, which is about 0.1 watt.

As this transfer occurred in 600 seconds, that is the total time that the power, due to this particular requirement, will be required. For the remainder of the period in the orbit, the power drain will be that required to overcome the friction in the bearings of the flywheels, and to correct any disturbances to the attitude of the satellite due to uneven distribution of meteor hits, aerodynamic forces, or rotational momentum remaining unabsorbed by the control rockets.

If the satellite has a larger angular momentum to be corrected than the power supplied to the flywheel installation can overcome in 600 seconds, due to one or a combination of the above causes, it merely means that the system will operate for a longer time. Essentially, for a given horsepower available, and a given ratio of satellite to flywheel moment of inertia, there is a definite satellite angular momentum which can be absorbed in unit time. If left to operate indefinitely, the installation could absorb any satellite angular momentum to the point where the velocity of the flywheel caused it to fly apart.

There is no foreseeable cause which would give the satellite such a large angular momentum to cause destruction of the flywheel, nor is there any persistent external force during the orbit which tends to rotate the satellite in a single given direction, and thus cause the flywheel to continuously accelerate.

As the auxiliary power plant which supplies power for radio and control, etc., in the orbit[2] produces on the order of 300 watts continuously, a non-continuous drain of the order required here is not objectionable.

References

[1] 'Aerodynamics, Gas Dynamics and Heat Transfer Problems of a Satellite Rocket', RA-15022, Project RAND, Douglas Aircraft Company, Inc., Feb. 1, 1947.

[2] 'Satellite Rocket Power Plant, RA-15027, Project RAND, Douglas Aircraft Company, Inc., Feb. 1, 1947.

[3] 'Communication and Observation Problems of a Satellite', RA-15028, Project RAND, Douglas Aircraft Company, Inc., Feb. 1, 1947.

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Standard Oil Company Esso Laboratories Elizabeth, New Jersey	Development Contract Officer Standard Oil Company Esso Laboratories, Box 243 Elizabeth, New Jersey	BUORD
University of Virginia Physics Department Charlottesville, Virginia Attn: Dr. J. W. Beams	Development Contract Officer University of Virginia Charlottesville, Virginia	BUORD
University of Wisconsin Madison, Wisconsin Attn: Dr. J.O. Hirschfelder	Inspector of Naval Material, 141 W. Jackson Blvd. Chicago 4, Illinois	BUORD
Westinghouse Electric Co. Essington, Pennsylvania	Bureau of Aeronautics Resident Representative Westinghouse Electric Corp. Essington, Pennsylvania	BUAER
Wright Aeronautical Corp. Woodridge, New Jersey	Bureau of Aeronautics Rep. Wright Aeronautical Corp. Woodridge, New Jersey	BUAER
Bethlehem Steel Corp. Shipbuilding Division Quincy 69, Mass. Attn: Mr. B. Fox	Supervisor of Shipbuilding, USN Quincy, Mass.	BUAER