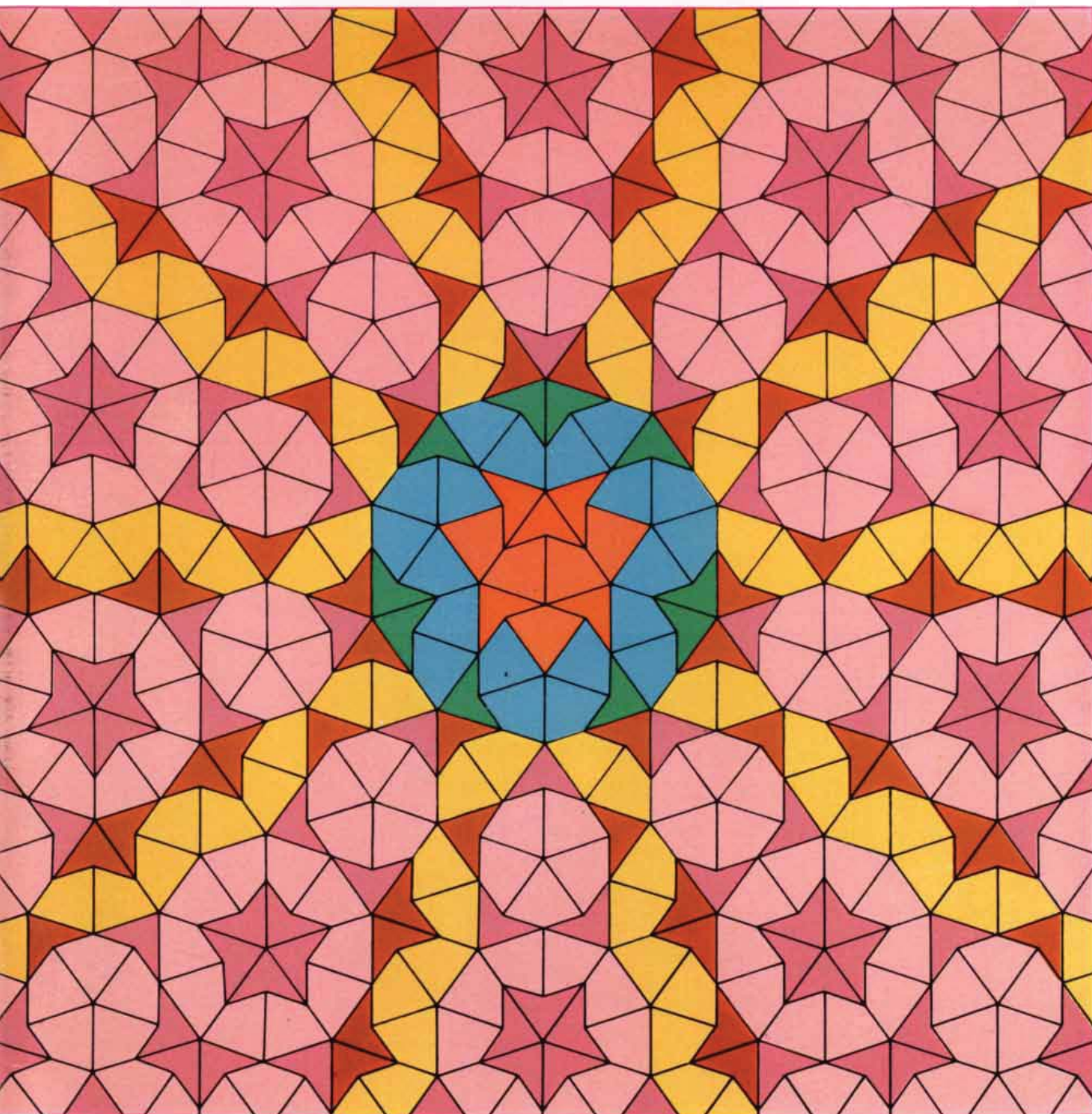


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The Quantum Mechanics of Black Holes

Black holes are often defined as areas from which nothing, not even light, can escape. There is good reason to believe, however, that particles can get out of them by "tunneling"

by S. W. Hawking

The first 30 years of this century saw the emergence of three theories that radically altered man's view of physics and of reality itself. Physicists are still trying to explore their implications and to fit them together. The three theories were the special theory of relativity (1905), the general theory of relativity (1915) and the theory of quantum mechanics (c. 1926). Albert Einstein was largely responsible for the first, was entirely responsible for the second and played a major role in the development of the third. Yet Einstein never accepted quantum mechanics because of its element of chance and uncertainty. His feelings were summed up in his often-quoted statement "God does not play dice." Most physicists, however, readily accepted both special relativity and quantum mechanics because they described effects that could be directly observed. General relativity, on the other hand, was largely ignored because it seemed too complicated mathematically, was not testable in the laboratory and was a purely classical theory that did not seem compatible with quantum mechanics. Thus general relativity remained in the doldrums for nearly 50 years.

The great extension of astronomical observations that began early in the 1960's brought about a revival of interest in the classical theory of general relativity because it seemed that many of the new phenomena that were being discovered, such as quasars, pulsars and compact X-ray sources, indicated the existence of very strong gravitational fields, fields that could be described only by general relativity. Quasars are star-like objects that must be many times brighter than entire galaxies if they are as distant as the reddening of their spectra indicates; pulsars are the rapidly blinking remnants of supernova explosions, believed to be ultradense neutron stars; compact X-ray sources, revealed

by instruments aboard space vehicles, may also be neutron stars or may be hypothetical objects of still higher density, namely black holes.

One of the problems facing physicists who sought to apply general relativity to these newly discovered or hypothetical objects was to make it compatible with quantum mechanics. Within the past few years there have been developments that give rise to the hope that before too long we shall have a fully consistent quantum theory of gravity, one that will agree with general relativity for macroscopic objects and will, one hopes, be free of the mathematical infinities that have long bedeviled other quantum field theories. These developments have to do with certain recently discovered quantum effects associated with black holes, which provide a remarkable connection between black holes and the laws of thermodynamics.

Let me describe briefly how a black hole might be created. Imagine a star with a mass 10 times that of the sun. During most of its lifetime of about a billion years the star will generate heat at its center by converting hydrogen into helium. The energy released will create sufficient pressure to support the star against its own gravity, giving rise to an object with a radius about five times the radius of the sun. The escape velocity from the surface of such a star would be about 1,000 kilometers per second. That is to say, an object fired vertically upward from the surface of the star with a velocity of less than 1,000 kilometers per second would be dragged back by the gravitational field of the star and would return to the surface, whereas an object with a velocity greater than that would escape to infinity.

When the star had exhausted its nuclear fuel, there would be nothing to maintain the outward pressure, and the star would begin to collapse because of its

own gravity. As the star shrank, the gravitational field at the surface would become stronger and the escape velocity would increase. By the time the radius had got down to 30 kilometers the escape velocity would have increased to 300,000 kilometers per second, the velocity of light. After that time any light emitted from the star would not be able to escape to infinity but would be dragged back by the gravitational field. According to the special theory of relativity nothing can travel faster than light, so that if light cannot escape, nothing else can either.

The result would be a black hole: a region of space-time from which it is not possible to escape to infinity. The boundary of the black hole is called the event horizon. It corresponds to a wave front of light from the star that just fails to escape to infinity but remains hovering at the Schwarzschild radius: $2GM/c^2$, where G is Newton's constant of gravity, M is the mass of the star and c is the velocity of light. For a star of about 10 solar masses the Schwarzschild radius is about 30 kilometers.

There is now fairly good observational evidence to suggest that black holes of about this size exist in double-star systems such as the X-ray source known as Cygnus X-1 [see "The Search for Black Holes," by Kip S. Thorne; SCIENTIFIC AMERICAN, December, 1974]. There might also be quite a number of very much smaller black holes scattered around the universe, formed not by the collapse of stars but by the collapse of highly compressed regions in the hot, dense medium that is believed to have existed shortly after the "big bang" in which the universe originated. Such "primordial" black holes are of greatest interest for the quantum effects I shall describe here. A black hole weighing a billion tons (about the mass of a mountain) would have a radius of about 10^{-13} centimeter (the size of a neutron or a

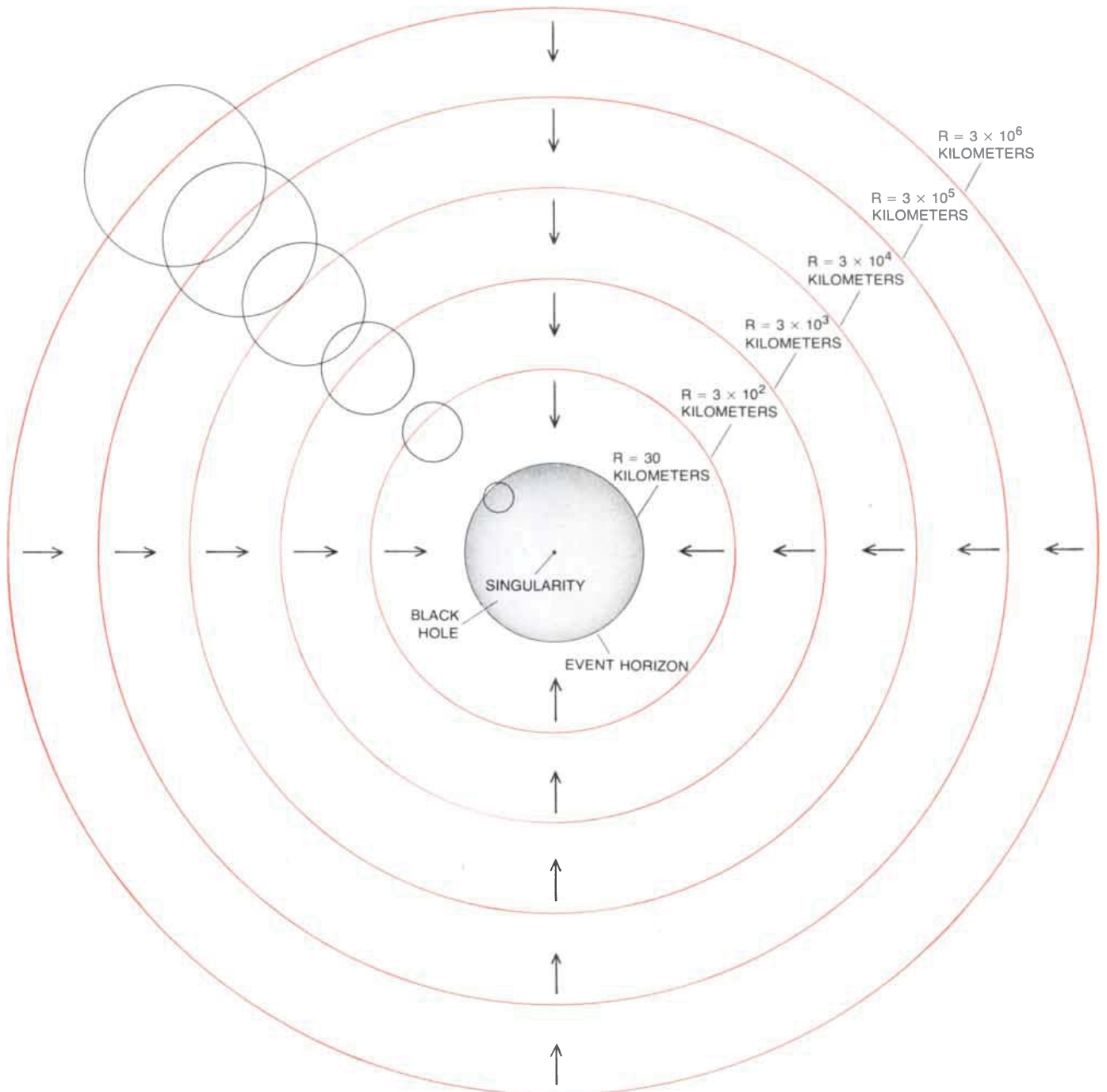
proton). It could be in orbit either around the sun or around the center of the galaxy.

The first hint that there might be a connection between black holes and thermodynamics came with the mathematical discovery in 1970 that the surface area of the event horizon, the boundary of a black hole, has the property that it always increases when addi-

tional matter or radiation falls into the black hole. Moreover, if two black holes collide and merge to form a single black hole, the area of the event horizon around the resulting black hole is greater than the sum of the areas of the event horizons around the original black holes. These properties suggest that there is a resemblance between the area of the event horizon of a black hole and

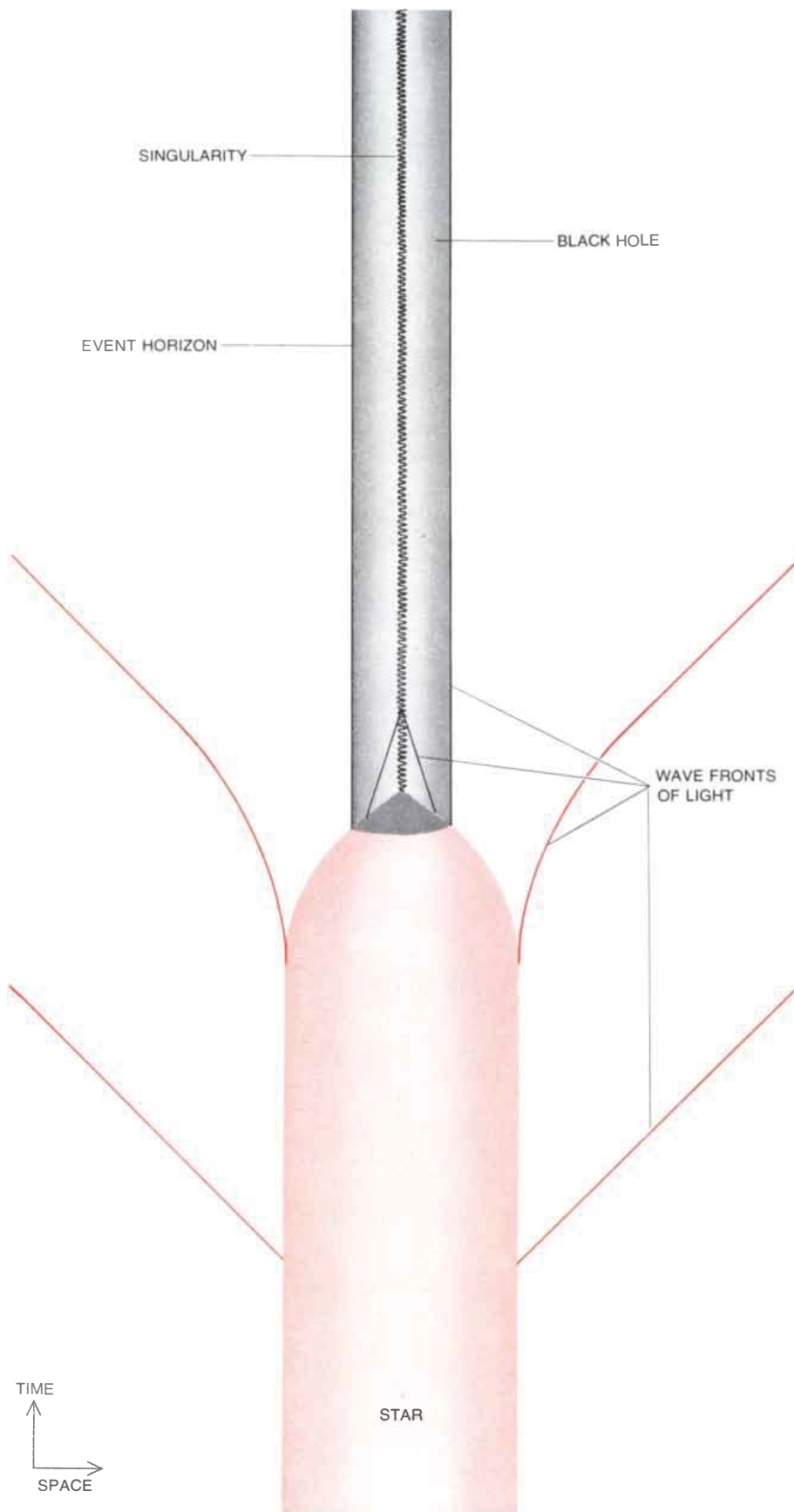
the concept of entropy in thermodynamics. Entropy can be regarded as a measure of the disorder of a system or, equivalently, as a lack of knowledge of its precise state. The famous second law of thermodynamics says that entropy always increases with time.

The analogy between the properties of black holes and the laws of thermodynamics has been extended by James M.



COLLAPSE OF STAR of 10 solar masses is depicted schematically from an original radius of three million kilometers (about five times the radius of the sun) to 30 kilometers, when it disappears within the "event horizon" that defines the outer limits of a black hole. The star continues to collapse to what is called a space-time singularity, about which the laws of physics are silent. The series of six small circles represents the wave fronts of light emitted from the successive sur-

faces an instant before the star had collapsed to the dimensions shown. Radii of the star and of the wave fronts are on a logarithmic scale. At each stage of collapse more of the wave front falls within the volume of the star as the escape velocity increases from 1,000 kilometers per second to 300,000 kilometers per second, the velocity of light. The final velocity is reached as the star disappears within the event horizon. No light emitted after that can ever reach outside observers.



GRAVITATIONAL COLLAPSE OF A STAR is depicted in a space-time diagram in which two of the three dimensions of space have been suppressed. The vertical dimension is time. When the radius of the star reaches a critical value, the Schwarzschild radius, the light emitted by the star can no longer escape but remains hovering at that radius, forming the event horizon, the boundary of the black hole. Inside black hole star continues collapse to a singularity.

Bardeen of the University of Washington, Brandon Carter, who is now at the Meudon Observatory, and me. The first law of thermodynamics says that a small change in the entropy of a system is accompanied by a proportional change in the energy of the system. The fact of proportionality is called the temperature of the system. Bardeen, Carter and I found a similar law relating the change in mass of a black hole to a change in the area of the event horizon. Here the factor of proportionality involves a quantity called the surface gravity, which is a measure of the strength of the gravitational field at the event horizon. If one accepts that the area of the event horizon is analogous to entropy, then it would seem that the surface gravity is analogous to temperature. The resemblance is strengthened by the fact that the surface gravity turns out to be the same at all points on the event horizon, just as the temperature is the same everywhere in a body at thermal equilibrium.

Although there is clearly a similarity between entropy and the area of the event horizon, it was not obvious to us how the area could be identified as the entropy of a black hole. What would be meant by the entropy of a black hole? The crucial suggestion was made in 1972 by Jacob D. Bekenstein, who was then a graduate student at Princeton University and is now at the University of the Negev in Israel. It goes like this. When a black hole is created by gravitational collapse, it rapidly settles down to a stationary state that is characterized by only three parameters: the mass, the angular momentum and the electric charge. Apart from these three properties the black hole preserves no other details of the object that collapsed. This conclusion, known as the theorem "A black hole has no hair," was proved by the combined work of Carter, Werner Israel of the University of Alberta, David C. Robinson of King's College, London, and me.

The no-hair theorem implies that a large amount of information is lost in a gravitational collapse. For example, the final black-hole state is independent of whether the body that collapsed was composed of matter or antimatter and whether it was spherical or highly irregular in shape. In other words, a black hole of a given mass, angular momentum and electric charge could have been formed by the collapse of any one of a large number of different configurations of matter. Indeed, if quantum effects are neglected, the number of configurations would be infinite, since the black hole could have been formed by the collapse of a cloud of an indefinitely large number of particles of indefinitely low mass.

The uncertainty principle of quantum

mechanics implies, however, that a particle of mass m behaves like a wave of wavelength h/mc , where h is Planck's constant (the small number 6.62×10^{-27} erg-second) and c is the velocity of light. In order for a cloud of particles to be able to collapse to form a black hole it would seem necessary for this wavelength to be smaller than the size of the black hole that would be formed. It therefore appears that the number of configurations that could form a black hole of a given mass, angular momentum and electric charge, although very large, may be finite. Bekenstein suggested that one could interpret the logarithm of this number as the entropy of a black hole. The logarithm of the number would be a measure of the amount of information that was irretrievably lost during the collapse through the event horizon when a black hole was created.

The apparently fatal flaw in Bekenstein's suggestion was that if a black hole has a finite entropy that is proportional to the area of its event horizon, it also ought to have a finite temperature, which would be proportional to its surface gravity. This would imply that a black hole could be in equilibrium with thermal radiation at some temperature other than zero. Yet according to classical concepts no such equilibrium is possible, since the black hole would absorb

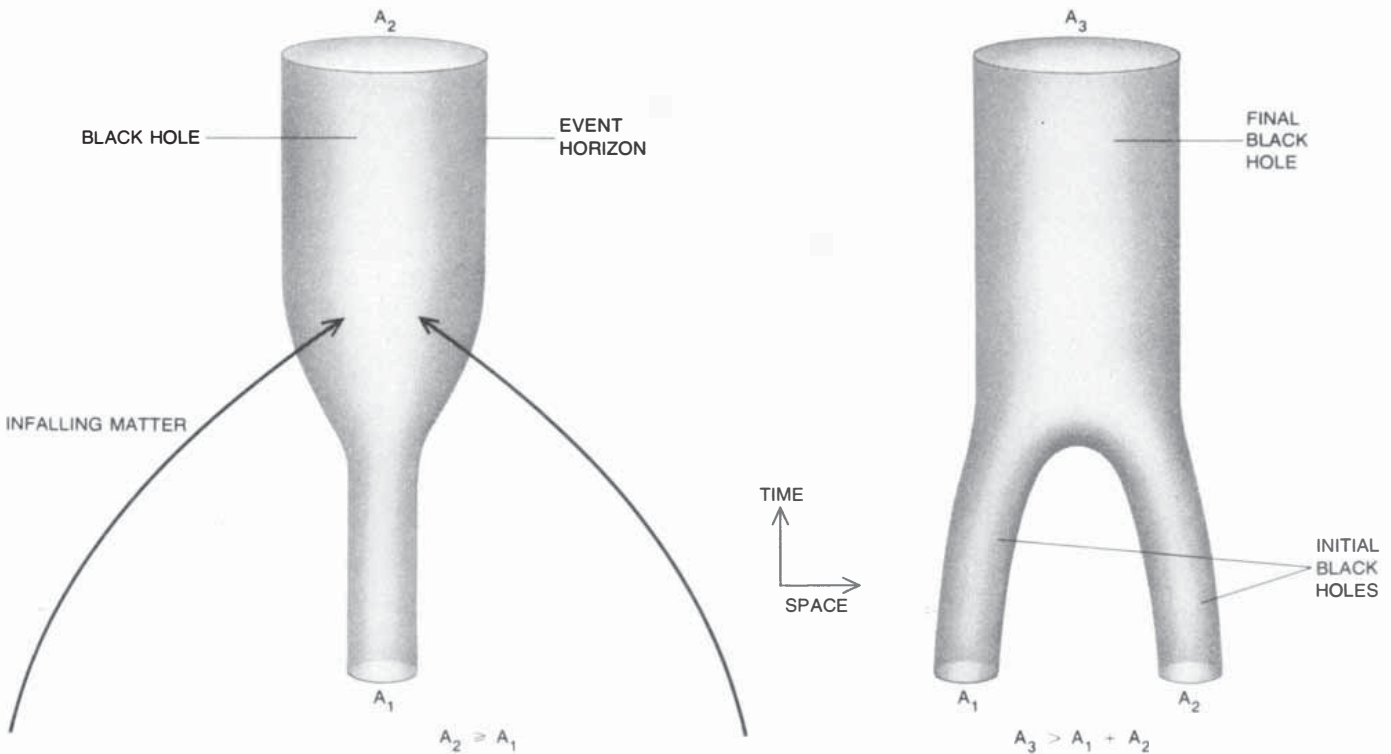
any thermal radiation that fell on it but by definition would not be able to emit anything in return.

This paradox remained until early in 1974, when I was investigating what the behavior of matter in the vicinity of a black hole would be according to quantum mechanics. To my great surprise I found that the black hole seemed to emit particles at a steady rate. Like everyone else at that time, I accepted the dictum that a black hole could not emit anything. I therefore put quite a lot of effort into trying to get rid of this embarrassing effect. It refused to go away, so that in the end I had to accept it. What finally convinced me it was a real physical process was that the outgoing particles have a spectrum that is precisely thermal: the black hole creates and emits particles and radiation just as if it were an ordinary hot body with a temperature that is proportional to the surface gravity and inversely proportional to the mass. This made Bekenstein's suggestion that a black hole had a finite entropy fully consistent, since it implied that a black hole could be in thermal equilibrium at some finite temperature other than zero.

Since that time the mathematical evidence that black holes can emit thermally has been confirmed by a number of other people with various different ap-

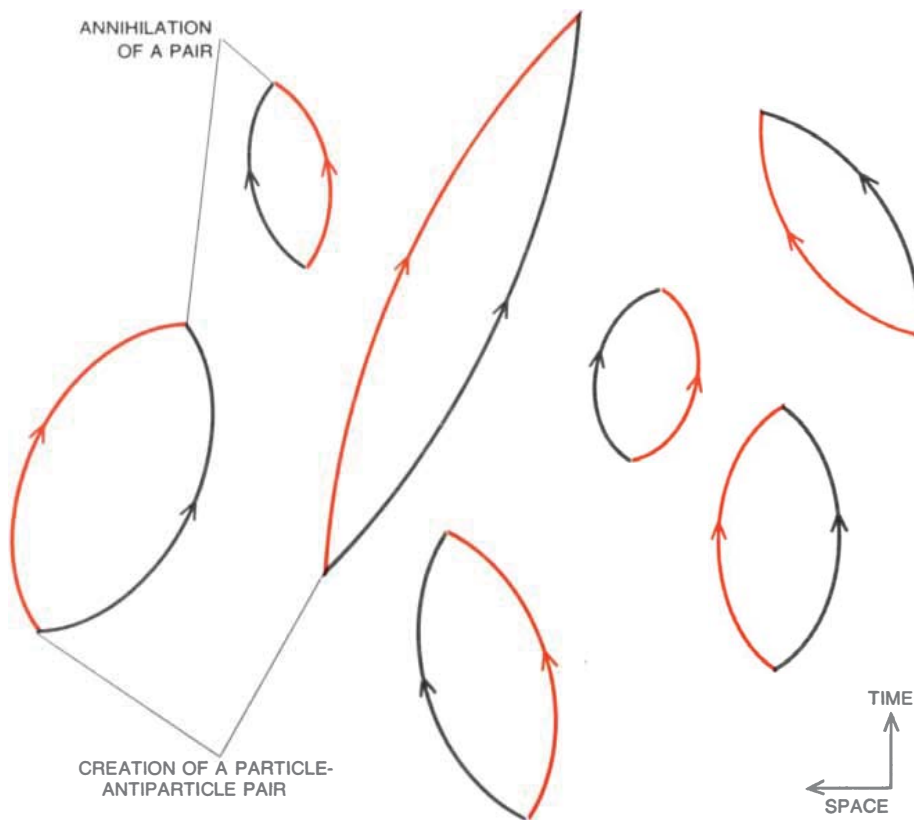
proaches. One way to understand the emission is as follows. Quantum mechanics implies that the whole of space is filled with pairs of "virtual" particles and antiparticles that are constantly materializing in pairs, separating and then coming together again and annihilating each other. These particles are called virtual because, unlike "real" particles, they cannot be observed directly with a particle detector. Their indirect effects can nonetheless be measured, and their existence has been confirmed by a small shift (the "Lamb shift") they produce in the spectrum of light from excited hydrogen atoms. Now, in the presence of a black hole one member of a pair of virtual particles may fall into the hole, leaving the other member without a partner with which to annihilate. The forsaken particle or antiparticle may fall into the black hole after its partner, but it may also escape to infinity, where it appears to be radiation emitted by the black hole.

Another way of looking at the process is to regard the member of the pair of particles that falls into the black hole—the antiparticle, say—as being really a particle that is traveling backward in time. Thus the antiparticle falling into the black hole can be regarded as a particle coming out of the black hole but traveling backward in time. When the particle reaches the point at which the

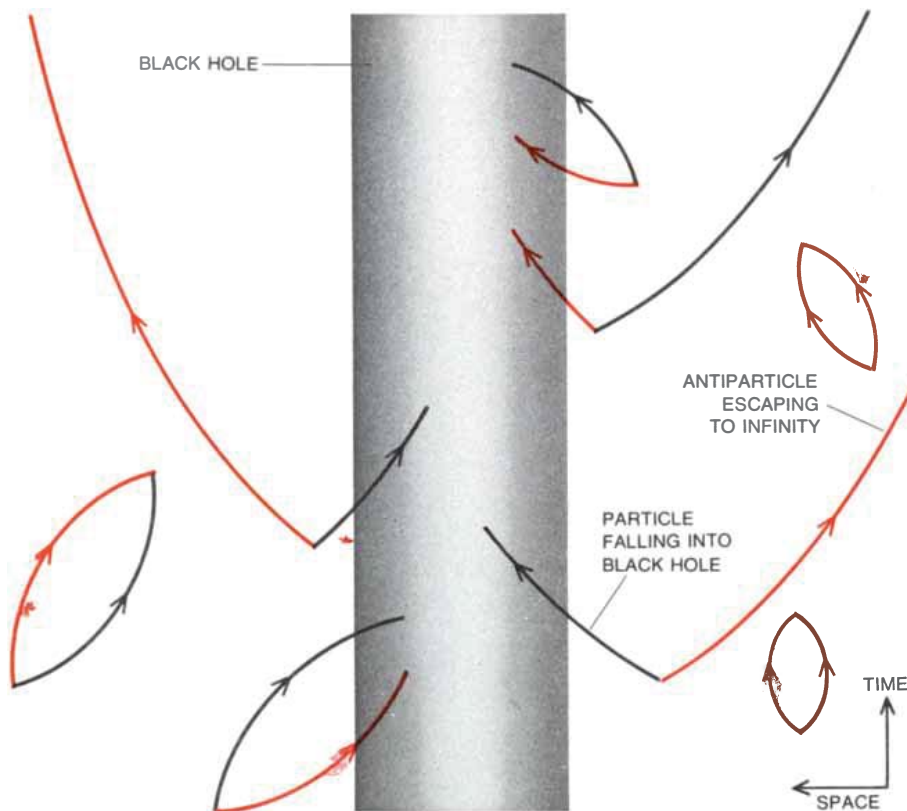


CERTAIN PROPERTIES OF BLACK HOLES suggest that there is a resemblance between the area of the event horizon of a black hole and the concept of entropy in thermodynamics. As matter and radiation continue to fall into a black hole (*space-time configuration at left*) the area of the cross section of the event horizon steadily in-

creases. If two black holes collide and merge (*configuration at right*), the area of the cross section of the event horizon of the resulting black hole is greater than the sum of the areas of the event horizons of the initial black holes. The second law of thermodynamics says that the entropy of an isolated system always increases with passage of time.



“EMPTY” SPACE-TIME is full of “virtual” pairs of particles (black) and antiparticles (color). Members of a pair come into existence simultaneously at a point in space-time, move apart and come together again, annihilating each other. They are called virtual because unlike “real” particles they cannot be detected directly. Their indirect effects can nonetheless be measured.



IN THE NEIGHBORHOOD OF A BLACK HOLE one member of a particle-antiparticle pair may fall into the black hole, leaving the other member of the pair without a partner with which to annihilate. If surviving member of pair does not follow its partner into black hole, it may escape to infinity. Thus black hole will appear to be emitting particles and antiparticles.

particle-antiparticle pair originally materialized, it is scattered by the gravitational field so that it travels forward in time.

Quantum mechanics has therefore allowed a particle to escape from inside a black hole, something that is not allowed in classical mechanics. There are, however, many other situations in atomic and nuclear physics where there is some kind of barrier that particles should not be able to penetrate on classical principles but that they are able to tunnel through on quantum-mechanical principles.

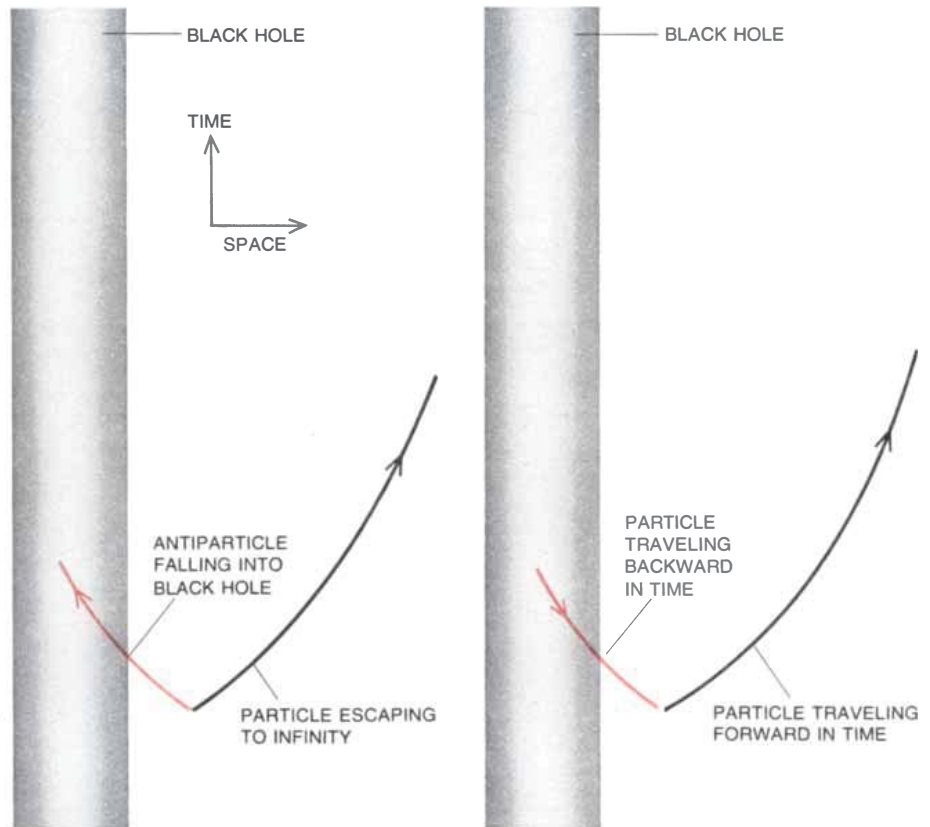
The thickness of the barrier around a black hole is proportional to the size of the black hole. This means that very few particles can escape from a black hole as large as the one hypothesized to exist in Cygnus X-1 but that particles can leak very rapidly out of smaller black holes. Detailed calculations show that the emitted particles have a thermal spectrum corresponding to a temperature that increases rapidly as the mass of the black hole decreases. For a black hole with the mass of the sun the temperature is only about a ten-millionth of a degree above absolute zero. The thermal radiation leaving a black hole with that temperature would be completely swamped by the general background of radiation in the universe. On the other hand, a black hole with a mass of only a billion tons, that is, a primordial black hole roughly the size of a proton, would have a temperature of some 120 billion degrees Kelvin, which corresponds to an energy of some 10 million electron volts. At such a temperature a black hole would be able to create electron-positron pairs and particles of zero mass, such as photons, neutrinos and gravitons (the presumed carriers of gravitational energy). A primordial black hole would release energy at the rate of 6,000 megawatts, equivalent to the output of six large nuclear power plants.

As a black hole emits particles its mass and size steadily decrease. This makes it easier for more particles to tunnel out, and so the emission will continue at an ever increasing rate until eventually the black hole radiates itself out of existence. In the long run every black hole in the universe will evaporate in this way. For large black holes, however, the time it will take is very long indeed: a black hole with the mass of the sun will last for about 10^{66} years. On the other hand, a primordial black hole should have almost completely evaporated in the 10 billion years that have elapsed since the big bang, the beginning of the universe as we know it. Such black holes should now be emitting hard gamma rays with an energy of about 100 million electron volts.

Calculations made by Don N. Page of the California Institute of Technology and me, based on measurements of the cosmic background of gamma radiation made by the satellite SAS-2, show that the average density of primordial black holes in the universe must be less than about 200 per cubic light-year. The local density in our galaxy could be a million times higher than this figure if primordial black holes were concentrated in the "halo" of galaxies—the thin cloud of rapidly moving stars in which each galaxy is embedded—rather than being uniformly distributed throughout the universe. This would imply that the primordial black hole closest to the earth is probably at least as far away as the planet Pluto.

The final stage of the evaporation of a black hole would proceed so rapidly that it would end in a tremendous explosion. How powerful this explosion would be depends on how many different species of elementary particles there are. If, as is now widely believed, all particles are made up of perhaps six different kinds of quarks, the final explosion would have an energy equivalent to about 10 million one-megaton hydrogen bombs. On the other hand, an alternative theory of elementary particles put forward by R. Hagedorn of the European Organization for Nuclear Research argues that there is an infinite number of elementary particles of higher and higher mass. As a black hole got smaller and hotter, it would emit a larger and larger number of different species of particles and would produce an explosion perhaps 100,000 times more powerful than the one calculated on the quark hypothesis. Hence the observation of a black-hole explosion would provide very important information on elementary particle physics, information that might not be available any other way.

A black-hole explosion would produce a massive outpouring of high-energy gamma rays. Although they might be observed by gamma-ray detectors on satellites or balloons, it would be difficult to fly a detector large enough to have a reasonable chance of intercepting a significant number of gamma-ray photons from one explosion. One possibility would be to employ a space shuttle to build a large gamma-ray detector in orbit. An easier and much cheaper alternative would be to let the earth's upper atmosphere serve as a detector. A high-energy gamma ray plunging into the atmosphere will create a shower of electron-positron pairs, which initially will be traveling through the atmosphere faster than light can. (Light is slowed down by interactions with the air molecules.) Thus the electrons and positrons will set up a kind of sonic boom, or shock wave, in the electromagnetic field. Such a shock wave, called Cerenkov ra-



ALTERNATIVE INTERPRETATIONS can explain the emission of particles by a black hole. One explanation (*left*) invokes the formation of a virtual particle-antiparticle pair, one member of which is trapped by the black hole as the other escapes. In another explanation (*right*) one can regard an antiparticle falling into a black hole as being a normal particle that is traveling backward in time out of the black hole. Once outside it is scattered by the gravitational field and converted into a particle traveling forward in time, which escapes to infinity.

diation, could be detected from the ground as a flash of visible light.

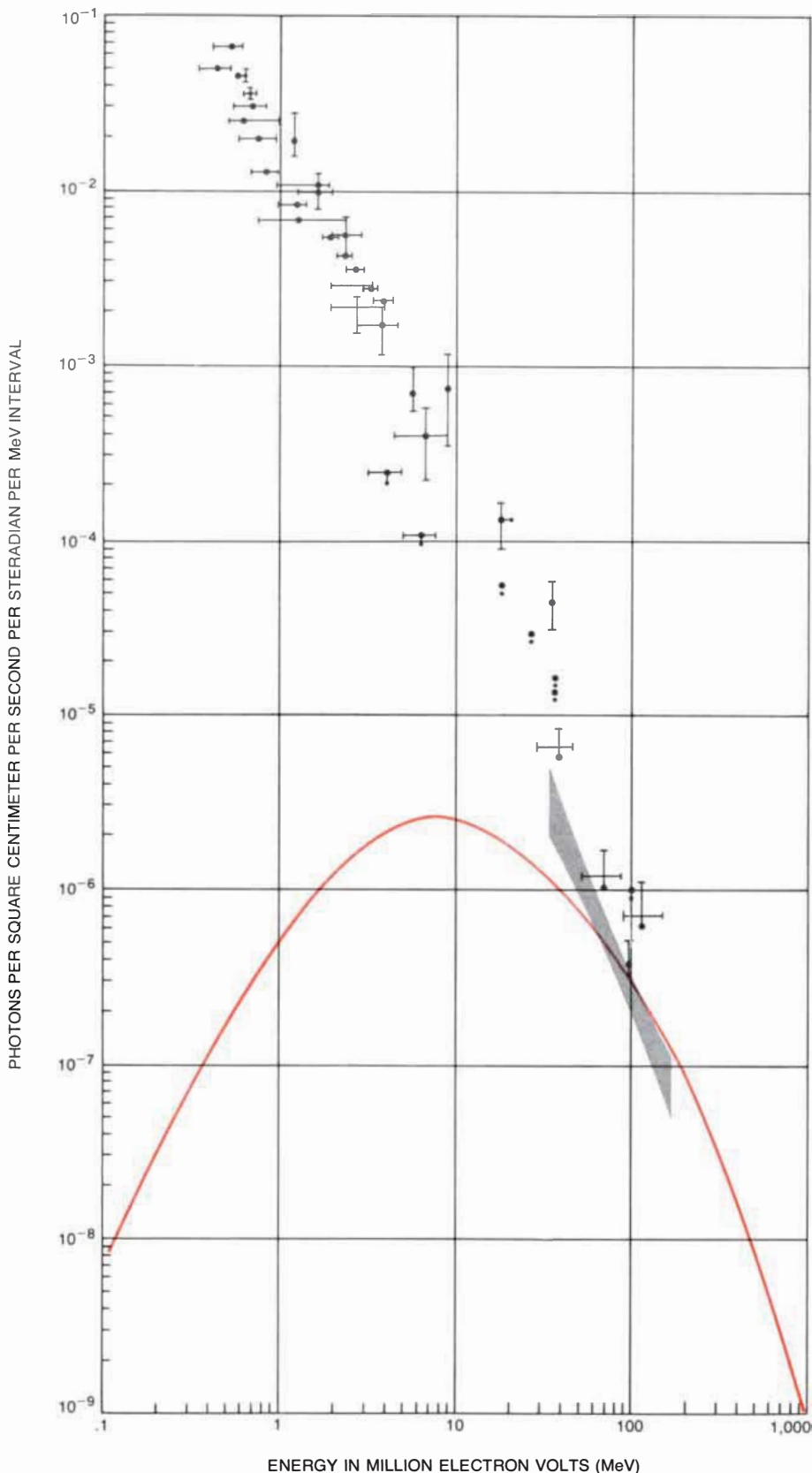
A preliminary experiment by Neil A. Porter and Trevor C. Weekes of University College, Dublin, indicates that if black holes explode the way Hagedorn's theory predicts, there are fewer than two black-hole explosions per cubic light-year per century in our region of the galaxy. This would imply that the density of primordial black holes is less than 100 million per cubic light-year. It should be possible to greatly increase the sensitivity of such observations. Even if they do not yield any positive evidence of primordial black holes, they will be very valuable. By placing a low upper limit on the density of such black holes, the observations will indicate that the early universe must have been very smooth and nonturbulent.

The big bang resembles a black-hole explosion but on a vastly larger scale. One therefore hopes that an understanding of how black holes create particles will lead to a similar understanding of how the big bang created everything in the universe. In a black hole matter collapses and is lost forever but new matter is created in its place. It

may therefore be that there was an earlier phase of the universe in which matter collapsed, to be re-created in the big bang.

If the matter that collapses to form a black hole has a net electric charge, the resulting black hole will carry the same charge. This means that the black hole will tend to attract those members of the virtual particle-antiparticle pairs that have the opposite charge and repel those that have a like charge. The black hole will therefore preferentially emit particles with charge of the same sign as itself and so will rapidly lose its charge. Similarly, if the collapsing matter has a net angular momentum, the resulting black hole will be rotating and will preferentially emit particles that carry away its angular momentum. The reason a black hole "remembers" the electric charge, angular momentum and mass of the matter that collapsed and "forgets" everything else is that these three quantities are coupled to long-range fields: in the case of charge the electromagnetic field and in the case of angular momentum and mass the gravitational field.

Experiments by Robert H. Dicke of Princeton University and Vladimir Braginsky of Moscow State University have



PRIMORDIAL BLACK HOLES, each about the size of an elementary particle and weighing about a billion tons, may have been formed in large numbers shortly after the big bang, the beginning of the universe as we know it. Such black holes would have a temperature of about 70 billion degrees Kelvin, corresponding to an energy of 10 million electron volts (MeV). The particles emitted at that energy would produce a diffuse spectrum of gamma rays detectable by satellites. The data points and the shaded region represent actual measurements of the diffuse gamma-ray spectrum in nearby space. The measurements indicate that the average density of such black holes in the universe must be less than about a million per cubic light-year. Solid curve is predicted spectrum from such a density of primordial black holes, based on plausible assumptions about the density of matter in universe and distribution of black holes.

indicated that there is no long-range field associated with the quantum property designated baryon number. (Baryons are the class of particles including the proton and the neutron.) Hence a black hole formed out of the collapse of a collection of baryons would forget its baryon number and radiate equal quantities of baryons and antibaryons. Therefore when the black hole disappeared, it would violate one of the most cherished laws of particle physics, the law of baryon conservation.

Although Bekenstein's hypothesis that black holes have a finite entropy requires for its consistency that black holes should radiate thermally, at first it seems a complete miracle that the detailed quantum-mechanical calculations of particle creation should give rise to emission with a thermal spectrum. The explanation is that the emitted particles tunnel out of the black hole from a region of which an external observer has no knowledge other than its mass, angular momentum and electric charge. This means that all combinations or configurations of emitted particles that have the same energy, angular momentum and electric charge are equally probable. Indeed, it is possible that the black hole could emit a television set or the works of Proust in 10 leather-bound volumes, but the number of configurations of particles that correspond to these exotic possibilities is vanishingly small. By far the largest number of configurations correspond to emission with a spectrum that is nearly thermal.

The emission from black holes has an added degree of uncertainty, or unpredictability, over and above that normally associated with quantum mechanics. In classical mechanics one can predict the results of measuring both the position and the velocity of a particle. In quantum mechanics the uncertainty principle says that only one of these measurements can be predicted; the observer can predict the result of measuring either the position or the velocity but not both. Alternatively he can predict the result of measuring one combination of position and velocity. Thus the observer's ability to make definite predictions is in effect cut in half. With black holes the situation is even worse. Since the particles emitted by a black hole come from a region of which the observer has very limited knowledge, he cannot definitely predict the position or the velocity of a particle or any combination of the two; all he can predict is the probabilities that certain particles will be emitted. It therefore seems that Einstein was doubly wrong when he said, "God does not play dice." Consideration of particle emission from black holes would seem to suggest that God not only plays dice but also sometimes throws them where they cannot be seen.