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Chapter 7

Casimir Coquilhat's Theory on Rocket Motion: The Rocket Equation Established in 1871!*

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Abstract

In the course of research on early rocketry, a so-far nearly unknown name cropped up only a few weeks ago, that of Belgian artillery general Casimir Coquilhat (1811–1890). A former director of the Antwerp arsenal, he actually wrote a note on 11 April 1871 titled "Trajectoire des fusées volantes dans le vide" ("Trajectory of Flying Rockets in Vacuum"). Mainly dedicated to war rockets use, it was published one year later in "Mémoires de la Société Royale des Sciences de Liège," a learned society he belonged to.

In this impressive 33-page work, Casimir Coquilhat analyzed the general motion of two kinds of war rockets, with and without lateral stick, for any launch angle and at any given time. In the second case, the simplest one, for a horizontal launch neglecting the effects of gravity, he established on page 13, among others, the following value for the velocity of a rocket:

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$$v = \frac{F}{m} \log \left(\frac{M}{M - mt} \right)$$

with F = thrust, m = mass flow rate, M = initial mass, and t = time.

Thus, the famous rocket equation established by Konstantin Tsiolkovsky on 10 May 1897 already had been found a full 26 years before, by Belgian Coquilhat, a rather stupendous discovery!

This chapter analyzes Casimir Coquilhat's fascinating theoretical work on the motion of a rocket, seemingly the most advanced one at the end of 19th century, a major step before Tsiolkovsky and Robert Esnault-Pelterie. It also will provide his biography.

Introduction

It sometimes happens, in the field of historical research, that seemingly insignificant facts are skipped, and that these omissions lead to partially or even totally erroneous conclusions but, still, widely accepted. The history of astronautics is no exception to that rule, and Casimir Coquilhat has just become the latest proof of it.

The so-called "action-reaction" rule, or Newton's third law, had been formulated by the latter as early as 1687, while the bases for the infinitesimal analysis had been established by Isaac Newton and Gottfried Leibniz* at the same time: the tools necessary to establish the equations related to rocket's motion thus already were available. But this theoretical work only happened to be completed late in the 19th century. It had been reputed for long that the great visionary of spaceflight, Konstantin Tsiolkovsky, had established these results in 1897. However, we now just have found that Casimir Coquilhat, a Belgian artillery general who had been a figure of the Independence Revolution in 1830–1831, already had solved this problem in 1871, and published it in 1872!

Context

In the years following Belgium's independence, the young Army officers had to translate foreign publications in order to manufacture military pyrotechnics, such as:²

^{*} Belgian surveyor De Sluze also worked on the infinitesimal theory. He died a few months after the publication of Leibniz' book on differential analysis (Cf. Sciences mathématiques et physiques au début du XIX° siècle, A. D. Quetelet, 1867).

- The user's guide written by Dutch Lieutenant-Colonel Sesseler, translated by Lieutenant-Colonel Timmerhans, who added an appendix. This *Manuel pour la confection des artifices de guerre* was published in Brussels in 1833.
- The book written by German Captain Moritz Meyer, translated in 1836 by Captain Hippert. Under the title of *Pyrotechnie raisonnée ou application* de la chimie aux artifices de guerre, it was published in Brussels and Paris.
- A second Moritz Meyer book, published and augmented by Hoffmann in 1840, and translated by Captain Neuens, who added his personal notes.³
 This Traité de pyrotechnie was published in Liège in 1843.

In the meantime, an *Ecole de pyrotechnie* had been created in Liège on 24 January 1841, according to royal decree N° 3938,⁴ "for handling gunpowder, manufacturing military pyrotechnical devices and introducing in this school all improvements that needs and scientific progress could make useful or mandatory." This school later was transferred to Antwerp in 1859.

During the 1850s, Captain Splingard, head of *Ecole de pyrotechnie*, proposed innovative solutions in the field of rocketry. He had the gunpowder compressed separately, then the grain "firmly" inserted in the rocket casing, thus allowing use of grains of complex geometry. Splingard thus tested military rockets with annular cores of 12 centimeters diameter. In his *Notice sur une nouvelle fusée de guerre*, he explained that the annular core introduced a new modification, allowing an increase of the rocket speed. He also had the idea of creating an additional core in the grain head. The combustion then could be arranged so as to switch in succession from the main core to the upper one. Splingard thus seems to have been the first to have proposed a double core rocket, allowing two thrust levels. He even thought in 1863 about bigger rockets, of a type only to see the light in the following century:

This loading method could be used for rockets of colossal dimensions... a one meter diameter rocket, a truly thundering arsenal, would yield its destructive power over a vast expanse of land, destroying all objects around its impact point.

Although lieutenant Nicaise demonstrated in *Considérations sur les Fusées de Guerre*⁸ the advantages of rockets in the country's defense, Belgian activities in the field of rocketry in fact did not go beyond experiments and theoretical analyses, no operational use being pursued by the Army. But among these analyses, those of General Coquilhat led to the earliest known establishment of the equations of the rocket motion!

Casimir Coquilhat

Casimir Erasme Coquilhat was born in Gent, in the Flemish area, on 4 October 1811, exactly 146 years before the launch of Sputnik 1!¹⁰ He was the son of Jacques-Philippe (1772–1840), a former French Napoleonian army soldier, who fought in the Austrian Low Countries (later Belgium), Holland, and Rhenania. Having emigrated to what became the Kingdom of Netherlands in 1815, he took up the corresponding citizenship, and was appointed chief teacher of rural colonies.

Coquilhat studied mathematics and physics at *Université de Liège*, graduating in differential/integral analysis and analytical mathematics. There, only 19 years old, he saw in August 1830 the uprising by the Belgians to gain their independence from the Netherlands. On August 30th, he was among the volunteers who attacked the Dutch army in the area of Sainte-Walburge. The Belgian revolutionaries next fought the Dutch soldiers, trapped inside the citadel of Liège, although outside help later allowed them to escape. Within weeks, the new state of Belgium was created, with its flag, national anthem, as well as an Army—fighting still was going on... Following his request of November 4th (he wanted to start right away as a lieutenant!), Coquilhat was enrolled on 27 December 1830 as an artillery cadet officer, however quickly becoming field artillery Second-Lieutenant on 15 May 1831. He took part in the decisive Ten Days Campaign against Holland, from August 2nd to 12th, 1831, stopping the Dutch in Cortessem on the 8th. Whereupon his influence and importance kept steadily



growing: he was promoted to Lieutenant on 1 May 1833, Captain in second on 30 July 1837, Captain on 4 June 1842, Major on 12 June 1851, Lieutenant-Colonel on 13 August 1858, and Colonel on 1 February 1865.

Figure 7–1: Casimir Coquilhat. Credit: http://www.ars-moriendi.be.

Also a teacher at *Ecole d'artillerie de Liège* since 2 November 1838, he was nominated deputy director of the *Fonderie Royale de Canons* in Liège on 13 February 1851,* deputy inspector for military weapons on 4 February 1859, commander for artillery weapons in Antwerp on 27 April 1859, and director of the military arsenal in Antwerp on 12 January 1867. On 13 December 1870, he finally was promoted as Major-General and Province Commander, still based in Antwerp. In 1783 he was asked to shift to the Western Flanders province, which he tried to oppose, even gun dueling against his successor in Antwerp (which led to his condemnation by a military court on 1 March 1875)! He retired on 21 December 1874.

While a director of the Arsenal, Coquilhat introduced new machines to improve the use of artillery equipment. He thus was the first to propose using recoil wedges for siege artillery, as well as manufacturing metallic cannon carriages.

Coquilhat was a recognized technical author. Among others, he was the author of Cours élémentaire sur la fabrication des bouches à feu en fonte et en bronze et des projectiles (Basic Course for Building Cast Iron and Bronze Ordnance Barrels and Projectiles), three volumes with figures, published in Liège in 1855, 1856, and 1858.

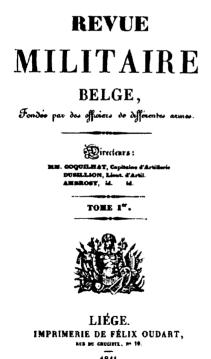
Figure 7–2: Coquilhat lecture book cover. Credit: F. Renard, Editeur, Liège.



Other publications can be mentioned: Expériences sur la résistance utile produite dans le forage des bouches à feu, in Liège in 1843 (Experiments on Friction when Drilling Ordnance Barrels); Evaluation de la quantité de travail absorbée par les frottements dans le forage des bouches à feu, in Liège in 1847 (Evaluation of Energy Absorbed by Friction when Drilling Ordnance Barrels); Expériences sur la résistance utile produite dans le forage du fer forgé, de la pierre calcaire et du grès, ainsi que dans le forage et le sciage du bois, in Tournay in 1850 (Experiments on Friction when Drilling Wrought Iron, Limestone

^{*} Liège, with its industrial Herstal district, already was well-known in the 18th century for weapons manufacturing. Created there in 1888, *Fabrique Nationale d'Armes de Guerre*, or FN Hertsal, collaborated with Browning. It later spun *FN Moteurs* during World War II, to build aircraft and rocket engine parts. It now is Techspace Aero, within Groupe Safran/SNECMA.

and Sandstone, as Well as in Drilling and Sawing Wood); Expériences faites à Ypres en 1850 sur la pénétration dans les terres de sondes en fer enfoncées par le choc d'un bélier, et application des fourneaux de mine cylindriques et horizontaux à l'ouverture des tranchées, in 1850 (Experiments in Ypres in 1850 on Ground Penetration of Iron Probes via a Ramming Device, and Use of Cylindrical and Horizontal Furnaces Filled with Landmines for Digging Trenches); Notes sur les projectiles creux et sur les bouches à feu, résistance à la rupture, tension des gaz, etc., in 1854 (Notes on Hollow Projectiles and on Ordnance Barrels, Fracture Strength, Gaz Pressure, etc.); Projet de deux canons à bombes pour l'artillerie de côte, du calibre 0,20 et 0,29, in 1854 (Project for a Bomb Cannon for Coastal Artillery, of 0.20 and 0.29 caliber); Percussions initiales produites sur les affûts dans le tir des bouches-à-feu, in Liège, in 3 parts, in 1863-1867 (Initial Ram Effect on Carriages in Ordnance Barrels Firings); Expériences sur la détermination pratique des moments d'inertie des canons, in 1864 (Experiments on Practical Measurement of Cannons Moments of Inertia); and the one which is of particular interest for us, Trajectoires des fusées volantes dans le vide, in 1872 (Rockets Trajectories in Vacuum). He also was one of the directors of *Revue* militaire belge in 1841, and member of the Société Royale Belge in Liège.



We incidentally recently found a picture of a gun manufactured in the 1850s in Liège, marked "Coquilhat & Digneffe," attributed to an Aimé Coquilhat, the owner of three patents on firearms. This gun was registered for testing between 1858 and 1866. His Coquilhat and Cie company existed in Liège up to 1888.

Interestingly, by November 1864, Coquilhat had been decorated by the Emperor of Russia, Commander of the Order of St Stanislas and Commander of the Order of Ste Anne. He received many other medals, in Belgium (Commander of the Order of Léopold, Volunteers Cross of 1830, Military Cross 1st Class, Commemorative Cross of 1856), in France Chevalier de la Légion d'Honneur, in Portugal Great Officer of the Order of Tower & Sword, in Prussia Officer of the Order of Red Eagle, in Turkey Chevalier of the Order of Medjidié.

Figure 7–3: *Revue Militaire Belge* cover (Coquilhat director). Credit: *Revue Militaire Belge*.

After he married the daughter of the division manager in the provincial government of Liège in 1852, he had a son, the following year, Camille. The latter became Vice-Governor of the Léopold II property in what later became Congo, where he was ahead of his time, hiring local people when he created the State Forces. In his honor, a town was named Coquilhatville (in 1936 it was part of the African network of Air France), renamed Mbandaka, the capital of the Equator Province, in 1966. He however died soon after his father in 1891, because of a tropical illness. A monument, now disappeared, was dedicated to him in Antwerp.

General Coquilhat died in Antwerp on 26 October 1890.

Analysis of the Document

Casimir Coquilhat's paper, "Trajectoires des fusées volantes dans le vide," is subtitled "Mouvement des fusées volantes: De la fusée et de sa force motrice" (Rockets Motion: About the Rocket and Its Thrust). He finished writing it on 11 April 1871, that is, three years before retirement. This was his last publication.

TRAJECTOIRES

DES

FUSÉES VOLANTES DANS LE VIDE

PAR

M. COQUILHAT,

General major ; offisier de l'ardre de Léopal ; describ major ; offisier de l'ardre de Léopal ; describ de la crois nommètica alive; checasier de l'ardre de Lion meritandais ; de l'aigle rouge de l'ardre de l'ardre de l'ardre de l'ardre de la Loue et l'Eprement de l'ardre de la Loue et l'Epremente de l'ardre de la Loue et l'Epremente de la commètre de la Société repair des secretes de Lifer

Figure 7-4: Coquilhat article title. Credit: Société Royale des Sciences de Liège.

His analysis of rocket motion is correctly based on the reaction principle: "Les gaz s'échappent par l'ouverture qu'offre le cartouche à la partie opposée à la tête. Leur réaction contre la partie restante de la composition détermine le mouvement de la fusée." (Gas are expelled from the gunpowder grain opening, opposite to the head. The reaction against the remaining part of the propellant determines the rocket motion). He stresses the mistake made by several of his predecessors: "Ce n'est donc pas la résistance ou l'appui que les gaz moteurs

rencontrent dans l'air, à leur sortie du cartouche, qui leur permet de propulser la fusée dans une direction opposée à leur propre mouvement." (It thus is not the resistance or the push of gases against the air, when expelled, which allow them propelling the rocket in an opposite direction to their own movement). Which brings him to clearly state: "Il en résulte que cet artifice ferait son ascension aussi bien et même mieux dans le vide que dans l'air." (As a consequence this ordnance will climb as well, and even better, in vacuum than in the air).

Coquilhat only analyzes the rocket motion in vacuum, as the drag equation progressively established by Da Vinci, Galileo, Mariotte, Bernouilli, and finally de Borda in 1763 was obviously unknown to him (it should also be noted that Le Bris' Barque Ailée, the first glider, already had flown in 1856, and that Lilienthal was soon to establish the polar curve in 1874): "Dans l'état actuel de la question, la forme de la trajectoire et les circonstances du mouvement dans l'air sont plutôt du domaine de l'empirisme." (In the present state of the question, the trajectory shape and the origins of motion in the air are rather empirical). On the other hand, his study duly takes into account the gravity losses, the projectile always staying close to the ground.

It also should be stressed that Coquilhat results only address the powered phase, and not the subsequent ballistic phase. This leads him to conclusions, which indeed are correct at burn-out, but wrong when one takes into account the complete flight down to the ground.

Coquilhat pre-supposes that the mass flow rate or "masse de composition brûlée dans l'unité de temps" (gunpowder mass burnt during the unit time) is constant. He explains that with a core shaped as a cone "dont la base est un cercle de même diamètre que celui intérieur du cartouche (...) le volume de la composition brûlée sera celui d'un cylindre, dont la base est égale à la section droite intérieure du cartouche et dont la longueur est celle du chemin parcouru par la surface comburante, et, comme cette longueur est proportionnelle au temps, la quantité des gaz produits sera également proportionnelle au temps." (the base of which is a circle with the same diameter as the inner one of the casing... the burnt volume of the grain will be that of a cylinder of a base equal to the inner section of the casing and a length equal to the distance traveled by the burning surface; since this length is proportional to time, the gas quantity also should be time linear). This already had been established by Buffon during the 18th century.¹²

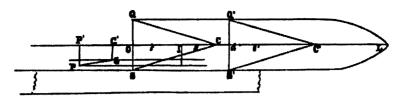


Figure 7–5: Figure from Coquilhat's paper (page 24): QCS is the initial core volume within the grain before ignition. Q'C'S' is the core volume after a given burn time t. The burned volume QCSS'C'Q' is equal to the cylindrical volume QSS'Q'. Credit: Société Royale des Sciences de Liège.

Coquilhat here does not mention the work of Splingard on the annular cores, which also provide constant rates.

Noting M the rocket initial mass, m the mass flow rate and t elapsed time since ignition, Coquilhat deduces that the gunpowder mass burnt away is at any moment mt, and thus the remaining rocket mass M-mt.

He then notes F the constant thrust, or "force motrice" of the rocket, and φ its acceleration or "force accélératrice" at a given elapsed time t, allowing:

$$\varphi = \frac{F}{M - mt}$$

On the basis of these hypotheses, Coquilhat analyzes two cases, whether the rocket stick is axial or lateral: "les fusées volantes forment deux classes, dont l'une a son centre de gravité sur l'axe de figure et affecte la forme symétrique par rapport à cet axe; l'autre classe possède des appendices extérieurs qui détruisent la symétrie de l'objet, et déterminent une position du centre de gravité en dehors de l'axe de figure." (rockets belong to two classes, one having their center of gravity on their main axis, thus impacting the symmetry with respect to this axis; the other one with external appendices destroying the object symmetry, thus putting the center of gravity outside the axis).

We will only consider here the first class, with the axial stick, a configuration which, according to Coquilhat, "se pratique pour les fusées de guerre perfectionnées." (is used for the advanced military rockets).

Noting θ the initial elevation of the rocket axis on the horizontal and g the gravity or "force accélératrice due à l'action de la pesanteur" (accelerating force due to the action of gravity), Coquilhat breaks down the rocket motion into the usual horizontal and vertical components.

A first integration of the acceleration then gives the horizontal and vertical velocities, supposing the latter are equal to zero at launch.

The second integration provides the horizontal and vertical positions, supposing the initial position is the reference for the trajectory.

But the most interesting result concerns the rocket speed at any given moment t:

$$v = \sqrt{\frac{F}{m} \log \left(\frac{M}{M - mt}\right) \left\{\frac{F}{m} \log \left(\frac{M}{M - mt}\right) - 2gt \sin \theta\right\} + g^2 t^2}$$

In his paper, Coquilhat finally declines the consequences of these results.

(1) "La force accélératrice [accélération] de la fusée croît constamment avec le temps t jusqu'au moment où toute la composition est brûlée" (The rocket acceleration constantly increases with time t until the gunpowder burns out). From that time "la force motrice [poussée] due à la combustion de la composition fusante n'existant plus, la fusée continuera à se mouvoir en vertu de la vitesse acquise et de l'action de la pesanteur, et la nouvelle partie de la trajectoire qu'elle décrira sera une courbe parabolique dont nous ne nous occuperons pas." (the thrust due to gunpowder combustion having disappeared, the rocket will continue its movement as a consequence of the speed achieved and the action of gravity, and the new trajectory then will be a parabola, that we will not delve into).

But he wrongly notes that "l'angle de tir (...) $\theta = 0^{\circ}$ donne la plus grande portée horizontale." (the $\theta = 0^{\circ}$ elevation gives the biggest horizontal range)

- (2) "Dans les fusées bien construites, le rapport λ/M doit être aussi fort que possible." (In the well-designed rockets, the λ/M ratio must be as high as possible), where λ is the gunpowder mass. The importance of the mass ratio is clearly stressed.
- (3) "Dans le cas où l'on aurait $\theta = 90^{\circ}$, (...) on a pour l'expression de la vitesse ascensionnelle dans un tir suivant la verticale" (in the case we would have $\theta = 90^{\circ}$, the vertical speed for a vertical ascent then becomes):

$$v = \frac{F}{m} \log \left(\frac{M}{M - mt} \right) - gt$$

(4) "Enfin, si la fusée était lancée horizontalement sur un plan également horizontal et s'il n'y avait ni obstacle, ni frottement, l'action de la pesanteur serait détruite par la résistance du plan et la vitesse de la fusée serait par l'hypothèse $\theta = 0$ et g = 0" (Finally, if the rocket were launched horizontally on an horizontal surface without any obstacle, the action of gravity would be canceled by the surface and the rocket speed would be, with $\theta = 0$ and g = 0):

$$v = \frac{F}{m} \log \left(\frac{M}{M - mt} \right)$$

This equation, resulting from the cancellation of gravity effects, is analogous to the well-known Tsiolkovsky one, the only difference being that the latter uses in the term preceding the logarithm, the gas exhaust speed, which is hard to measure. Meanwhile, with Coquilhat, the term is shown as a ratio between two scalars, which could be measured at the time:

- the thrust, for which benches already had been developed,*
- the mass flow rate, obtained as the ratio between the gunpowder mass and the burn duration.

In this context, it is worth noting that famous Russian Konstantin Konstantinov (he now would be Ukrainian), who had close connections with France, spoke and wrote in French, happened to be a significant intermediary, as he made the first and decisive step in the direction of the rocket equation when, in 1857, he wrote: "the moment quantity given to the rocket is, at any time, equal to the moment quantity of the expelled gas." But he did not perform the final step, the differential analysis giving the value of the rocket speed as a function of the mass ratio.

Conclusion

When it appeared, Coquilhat's work on the motion of rockets was the most elaborate in the world. It however seems to have stayed confidential—strangely enough, like Tsiolkovsky's work later, which only started being known about 1927—maybe because it coincided with the disappearance of the military rocket, because of the progress of conventional artillery (rifling, breach loading).

Maybe also, being known as a gun and cannon specialist, Coquilhat was not expected to suddenly publish something about rockets, even more on a theoretical level, while he always had been a practitioner and lately more of a manager. We have not found so far the reasons for this late interest.

It is interesting at this stage to remember that paper¹⁴ presented during the second IAF History of Astronautics Symposium in 1968 revealed that the famous "Rocket Equation" of Tsiolkovsky actually had been established as early as 1897, and not 1903 as previously thought.

Now, 40 years later, the incredible discovery has been made that the rocket equation actually was established 137 years ago on 11 April 1871 by Belgian General Coquilhat, a full 26 years before the first great space pioneer, Russian Tsiolkovsky, and with an even more general formulation!

^{*} The 1846 research program of *Ecole Centrale de Pyrotechnie* in Metz included measuring military rocket thrust by using the blades of a recording dynamometer.¹⁵

This of course does not detract from the fact that Tsiolkovsky's astronautical work was immense, while Coquilhat's rocket motion analysis was his only rocket work ever. But a pillar of astronautics now has found a new and unlikely father in Belgium.

Acknowledgements

The authors wish to thank Michaël Massy for his help in finding the original documents.

Appendix

The discovery of the seminal Coquilhat paper is of such interest that it is worth detailing it. Jean-Jacques Serra, in the course of his systematic searches on pre-astronautics rockets, went into books.google.com with the key word, often used in the old times, of "fusées volantes". In the beginning of 2008, he found the following reference in Mémoires de la Société Royale des Sciences de Liège (SRSL):

- Tome 5, 1874. Dosage de l'acide carbonique, par Kuffferschlaeger. - Insectes recueillis au Japon par G. Lewis (1869-71); Elatérides, par E. Candèze. - Intégration des équations aux dérivés partiels des deux premiers ordres, par J. Graindorge. - Essai sur les Antarctia Dejean, par J. Putzeys. - Trajectoires des fusées volantes dans le vide, par Coqualitat. - Cléonides, par Chevrolat. - Arabéides nouveaux du midi de l'Europe, avec 3 planches, par E. Simon. 1 vol. in 8.

He then established a contact, via a space enthusiast web forum, with Belgian Michaël Massy, who directly obtained the article from SRSL. Interestingly, this paper had been duly referenced at the time of its publication in several countries: France, Great Britain (Royal Society, Catalogue of Scientific Papers) and Prussia (Akademie für Wissenschaften, Jahrbuch über die Fortschritte der Mathematik). He even found that this paper had been referenced since 1990 in The Golden Age of Rocketry book, written by American historian Frank Winter!

On a d'ailleurs pour la vitesse à un instant quelconque t,

$$v = \sqrt{\frac{F}{m} \log \left(\frac{M}{M - mt}\right) \left\{\frac{F}{m} \log \left(\frac{M}{M - mt}\right) - 2gt \sin \theta\right\} + g^2 t^2}.$$
 (7)

Enfin le coefficient angulaire de la tangente à la trajectoire a pour valeur

$$\frac{dy}{dx} = \lg 2 - \frac{mgt}{F \cos \theta \log \left(\frac{M}{M - mt}\right)} (8)$$

Extract from page 7 of Coquilhat's rocket paper.

Dans le cas où l'on aurait 0 = 90°, la quantité sous le radical de l'équation (7) devient un carré parfait et l'on a pour l'expression de la vitesse ascensionnelle dans un tir suivant la verticale.

$$v = \frac{F}{m} \log \left(\frac{M}{M - mt} \right) - gt.$$

Dans le cas où la fusée serait dirigée verticalement du haut en bas, on aurait

$$v = \frac{F}{m} \log \left(\frac{M}{M - mt} \right) + gt.$$

Enfin, si la fusée était lancée horizontalement sur un plan également horizontal et s'il n'y avait ni obstacle, ni frottement, l'action de la pesanteur scrait détruite par la résistance du plan et la vitesse de la fusée serait par l'hypothèse:

$$0 = 0 \quad \text{ct} \quad g = 0,$$

$$\mathbf{c} = \frac{\mathbf{F}}{\mathbf{M}} \log \frac{\mathbf{M}}{\mathbf{M} - mt}.$$

Extract from page 13 of Coquilhat's rocket paper.

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